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Remarks on the theory of interaction

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Various possibilities for the interaction transition of Fermi or Bose particles are discussed, and a comparison between the Dirac and Fermi models for particle creation is outlined.

The development of the theory of β-rays has obviously now entered a new arena. The original Fermi form for the coupling of a heavy particle with the fields of electrons and neutrinos does not suffice to reproduce the correct form for the spectrum, while the Ansatz of Konopinski-Uhlenbeck (K-U) regarding the first derivative of the neutrino wave functions seems to be well-confirmed empirically, but encounters formal difficulties when one carries out second quantization. The entire complex of questions about the derivation of the nuclear forces, as well as the magnetic moment of heavy particles and the Heisenberg theory of cosmic showers, admits only a qualitative treatment. On the other hand, isolated results in the entire theory of β-radiation suggest the opinion that very many truths will be expressed there, and it seems desirable to also propose some special cases for discussion. In what follows, we shall discuss some simple cases of the interaction transition, in the sense of the theory of nuclear β-forces.

1. In order to better glimpse the influence of statistics, we consider, above all, the interaction transition of Bose particles. If the transition takes place between isolated particles that satisfy the Bose statistics then we have, in the simplest case, the d'Alembert equation for the scalar potential of the electromagnetic field:

$$\Box\varphi = 0,$$

which yields the Coulomb law for the interaction in the event that the coupling of the particle with the field of the – longitudinal – photons is given by $U = e\varphi$. However, when the wave function of the interacting particle satisfies the scalar, second-order, relativistic equation:

$$(\Box + k_0^2)\psi = 0 \tag{1}$$

(where $k_0 = m_0c / h$) then one show that the interaction law between two particles, as a solution of equation (1), must have the form $e^{-k_0 r} / r$. One can then easily see that either the purely classical Fermi method of elimination for the longitudinal part ([1]) or the quantum electrodynamical method for the calculation of the second approximation will both reproduce the Green function for the field equation in question.

If we apply – e.g. – the Dirac method of quantum electrodynamics then we get the interaction law in the form:

$$V = U_1 \cdot \frac{U_2}{S} + U_2 \cdot \frac{U_1}{S}, \tag{2}$$

where S means the operator of the Schrödinger equation and U means the interaction energy of a particle with the field. We represent the latter in, perhaps, the form:

$$U = e(\psi_1 + \psi_1^+) + e(\psi_2 + \psi_2^+),$$

with the usual Fourier decomposition for ψ:

$$\psi = \frac{1}{(2\pi)^{3/2}} \int (dk) \{a(k)\, e^{-icKt + i(kr)} + b^+(k)\, e^{+icKt + i(kr)}\}, \tag{2a}$$

$$K^2 = k^2 + k_0^2 .$$

If one assumes the Bose commutation relations (C.R.):

$$a(k)\, a^+(k') - a^+(k')\, (a(k) = \frac{hc}{2K}\, \delta(k - k'), \qquad \text{etc.} \tag{2b}$$

and the initial condition of the absence of particles of both kinds – i.e., $a^+(k)\, a(k) = 0$ – then we obtain, after an integration:

$$V = \frac{\text{const.}}{r} \int_0^\infty \frac{k\, dk}{k^2 + k_0^2} \sin kr . \tag{3}$$

For $k_0 = 0$, the Coulomb law is true, whereas here one obtains:

$$V = \frac{\text{const.}}{r} e^{-k_0 r} . \tag{4}$$

Despite the fact that the well-known bilinear decomposition ([2]) of the Green function includes the eigenvalue λ_k in the denominator, which is equal to K^2 here, in formula (2), the operator S appears in the denominator, which fails to be proportional to K, so the C.R. add a second K. In this way, one sees intuitively the return of the Green function as a result of the truly complicated quantum-electrodynamical calculation. Recently, Oppenheimer and Serber, building upon an old argument of Yukawa, have proposed the

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([1]) See Heitler's *Theory of Radiation*, pp. 51.
([2]) Cf., Courant-Hilbert, *Methoden der mathematischen Physik*, III, § 5.

law of the form (4) as the nuclear force between neutrons and protons. However, one must emphasize that this potential is realized for Bose particles, which is perhaps notably not the case for interacting hypothetical "semi-heavy" electrons ($m > m_{el}$).

If we deal with a true scalar ψ then on the grounds of relativistic covariance we actually must not consider the field in the form $e\psi$, but perhaps in the form:

$$U = \sum_{s=1,2} e_s \left(\frac{\partial \psi}{\partial t} + \frac{\partial \psi^+}{\partial t} \right)_s , \tag{5}$$

since it should represent the time component of a vector. We then arrive at an interaction of the form r^{-3}, with a factor that is not, however, equal to zero. The individual quanta of a scalar field are thus incapable of realizing an interaction in the second approximation, which agrees with the requirement of gauge invariance.

In the case of the interaction of two Bose particles, it is natural to take the following expression for the fourth component of the mixed current of the scalar, relativistic second-order equations:

$$U = g' \left(\varphi_2^+ \frac{\partial \varphi_1}{\partial t} - \varphi_1^+ \frac{\partial \varphi_2}{\partial t} \right) + \text{c.c.} \tag{6}$$

to represent the coupling of the interacting particles with the field of the pair of Bose particles. It is well-known that the scalar, relativistic equation admits only symmetric statistics. If one now employs the quantum-electrodynamical perturbation calculation then one obtains an interaction law of the form:

$$V = \text{const.} \cdot r^{-5} \tag{7}$$

in the second approximation. Thus, one gets the same dependency upon r as one does in the interaction of two Fermi particles – e.g., an electron and a neutrino – in the event that the coupling of a heavy particle with the field is given by $U = g\psi_e^+\psi_n$. Despite the fact that the coupling (6) includes derivatives, we obtain an r^{-5} law, and not an r^{-7} law, which would follow from the K-U formula $U = g'\psi_e^+\partial\psi_n/\partial t$. The basis for this lies in the compensation of the Konopinski-Uhlenbeck factor: $E_{\text{neutrino}} = hcK$ in the numerator by precisely the same factor in the denominator as a result of the Bose commutation relations for the Fourier coefficients (2b); the factor K^{-1} is indeed absent in the Fermi C.R. The calculations that led to (7) are entirely analogous to the ones in the case of the Fermi theory, and give rise to almost identical integrals that one must evaluate by means of perhaps an auxiliary factor $e^{-\alpha r}$ ($\alpha \to 0$).

2. If the interacting particles satisfy Fermi statistics then relativistic covariance already demands the introduction of a second particle, since a single electron cannot radiate. In fact, one can certainly not construct a vector that is linear in ψ with the help of a single spinor, and in this way define the coupling with the field of the electrons as the fourth component of such a vector. Despite the fact that one can be inclined to examine this situation more closely by means of the introduction of a differential spinor operator ∂_λ, it is known that an argument based upon conservation laws, independently of the more formal grounds that were cited, leads to the hypothesis of the simultaneous emission of neutrinos and electrons.

As far as the derivation of the nuclear forces is concerned, which has already been discussed by many authors, here we remark only that the simplest, and certainly the most intuitive, method consists in the introduction of the doubled Fourier decomposition:

$$\psi \to \psi_{\text{el.}} + \psi_{\text{pos.}}\,; \qquad \varphi \to \varphi_{\text{neutr.}} + \varphi_{\text{antineutr.}}\,, \tag{8}$$

or, more precisely:

$$\psi_s = (2\pi)^{-3/2} \int (dk)\, \{a^s(k)\, e^{-icKt + i(kr)} + b^{s+}(k)\, e^{+icKt + i(kr)}\},$$

where $a^s(k)$ describes the electrons and $b^s(k)$ describes the positrons, and analogously, perhaps $c^s(k)$ could describe neutrinos, while $d^s(k)$ describes anti-neutrinos. Such decompositions were already applied advantageously in second quantization and the examination of the neutrino theory of light ([1]). In the calculations of the second approximation, one now chooses, quite intuitively, the terms that correspond to the radiation of an electron-anti-neutrino pair by the first heavy particle and its absorption by the second heavy particle. One distinguishes the terms that correspond to the radiation and absorption of a positron-neutrino pair analogously. If one thus considers the initial condition of the absence of light particles then one is left with only terms of the form aa^+, bb^+, cc^+, dd^+.

The summation over two possible spin states is easy to carry out without the Casimir auxiliary method, and in that way, with no mention of the states of negative energy, we come to the following expression for the interaction:

$$V = \frac{(2\pi)g^2}{(2\pi)^6} Q_1 Q_2^+ \iint \frac{(dk)(dl)}{k+l} [1 - \cos(kl)\{e^{-i(\overline{k+l})\vec{r}} + e^{+i(\overline{k+l})\vec{r}}\} + \text{c.c.} \tag{9}$$

Therefore, the coupling of heavy particles with the field of the electrons and neutrinos would be exhibited in the Fermi form:

$$U = \sum_{s=1,2} g_s (\psi^+ \varphi Q + c.c.)_s\,, \tag{10}$$

where Q denotes the Heisenberg operators of the neutron-proton transition and $s = 1, 2$ enumerates the heavy particles 1 and 2.

The complex conjugate (c.c.) terms in (10) correspond to the positrons “anti”-decay, and yield the same result in our approximation as the electron terms, which is also immediately understandable in terms of physics. The integration of (10) yields the known result ([2]):

([1]) D. Iwanenko and A. Sokolow, Sow. Phys. **11** (1937), 590. A. Sokolow, *ibidem* **12** (1937), 148.
([2]) Cf., Ig. Tamm, Sow. Phys. **10** (1937), 567; C. Weizsäcker, ZS. f. Phys. **102** (1936), 572.

$$V = \frac{g^2}{hcr^5} \cdot \frac{1}{2\pi^2} (Q_1 Q_2^+ + Q_2 Q_1^+) . \tag{11}$$

The earlier result ([1]) was derived by neglecting the factor [1 – cos (*kl*)], which originates in the spins, and thus agrees with the spinless part of the total result (part *d* of Weizsäcker).

Since the interaction (11) is know to yield nuclear forces that are much too weak, one must seek to calculate using K-U Ansätze about derivatives in the coupling (10). The introduction of the first derivative is indeed sufficient for the effects of first, but not second, order, since the latter require the introduction of derivatives with total order three. The assumption of radiation of not two, but several, particles also leads to the same result, whereby the next minimal number will be equal to four.

A coupling of the form:

$$U \sim g' \psi \varphi^3, \tag{12}$$

instead of (10), would yield an interaction of the form r^{-11}, and is, in many regards, equivalent to the introduction of the derivatives of third order for the neutrino function. Our hypothesis is also apparently equivalent to the recent attempt of Nagendra Nath ([2]), who, in fact, assumed the primary radiation of only one pair, and then suggested grounds for the secondary intensive radiation of neutrinos by the β-electron.

From the theoretical standpoint, it would be more satisfying to regard all of these generalizations of the Fermi Ansatz as the initial terms in a development in some new constant. Recently, Richardson ([3]), among others, has successfully employed the sum of the Fermi and K-U Ansätze to the empirical interpretation of the β-spectrum, while the possibility of introducing the second constant and a double-terms formula was also discussed by Tamm ([4]). In it, the initial terms must give the form of the spectrum, while the higher terms (with higher derivatives, or the terms that correspond to several particles) must give the nuclear forces in the correct order of magnitude. The situation seems to us to be broadly analogous to the relationship of the Born nonlinear theory to Maxwell's theory. The usual theory of radiation, which starts with the coupling of the electromagnetic field of the form $U = e\varphi$, admits the radiation of only photons in the first approximation (for the sake of brevity, we shall not distinguish between transversal and longitudinal photons here). However, the Born theory regards the Maxwell potential as only the first term in a development in the quantity r_0^4, and, in turn, admits the radiation of many photons in the first approximation of perturbation theory (although in a higher approximation the decomposition into r_0^4 or $1/b^2$).

It might also be useful to consider sums of the form:

$$U \sim g_1 \psi \varphi + g_2 \psi \frac{\partial \varphi}{\partial t} + \cdots + g_{nm} \frac{\partial^m \psi}{\partial t^m} \frac{\partial^n \varphi}{\partial t^n} \tag{13}$$

([1]) D. Iwanenko and A. Sokolow, ZS. f. Phys. **102** (1936), 119.
([2]) Nagendra Nath, Nature **140** (1937), 501.
([3]) H. O. W. Richardson, Proc. Roy. Soc. A **161** (1937), 447.
([4]) Ig. Tamm, *loc. cit.*

or

$$U \sim g_1 \psi \varphi + g_2 \psi \varphi^3 + \ldots \tag{13a}$$

instead of the primary Fermi coupling (11), and, in turn, introduce the closed forms of the decompositions (13) from the outset, whereby one thinks of the latter as evolving from a single constant (perhaps a new specific length or a "maximal density" of the light particles).

3. In conclusion, we would like to make some remarks on the comparison between the Dirac and Fermi methods of describing the creation of a particle. The matrix element of the "immediate" radiation of the pair in the Fermi theory:

$$g' \int \chi_m^+ (\psi^+ \varphi) \chi_n \, d\tau \tag{14}$$

(where χ denotes the wave functions of the heavy particles) and the matrix element for the creation of the electron-pair under the influence of a γ-photon (internal conversion) according to Dirac:

$$e \int \psi^+_{\mathrm{el}_n} (vA) \psi_{\mathrm{pos}_m} \, d\tau \tag{15}$$

differ in appearance in many respects. However, if one introduces the second quantization for all particles then the difference between radiating and radiated particles disappears, since all particles experience a certain transition. Thus, it is preferable to apply the double decomposition (8) that was carried out above for ψ; formula (15) then seems to be precisely identical with the usual expression for the radiation (absorption) of light under the transition $m \to n$. All that then remains is the difference between the radiation of *two* particles in (14) and *one* particle (a photon) in (15). If one then takes the viewpoint of the neutrino theory of light then one can replace A with a bilinear expression in the neutrino wave function. However, conversely, one can also arrive at the formula (15) by starting with the Fermi expression (14).

We have already mentioned above that the Konopinski-Uhlenbeck introduction of derivatives corresponds, in a sense, to the assumption of the radiation of many particles, and in this way, so to speak, leads to an increase in the independence between particles. The introduction of inverse operators, and therefore of integrals, instead of differentials, must reduce the effective number of equivalent independent particles (two, in the Fermi case), or else introduce a certain degree of dependency or "coherence" between the particles. If one applies, e.g., the inverse d'Alembertian operator $\Box^{-1}$ to the Fermi Ansatz (11) then one does not change the covariance character of the Fermi coupling as the temporal component of the vector:

$$U = g' \chi^+ \frac{\psi^+ \varphi}{\Box} \chi \to g' \chi^+ \int d\tau \frac{\psi^+ \varphi}{r} \cdot \chi_{t'=t-r/c} \,. \tag{16}$$

Such a coupling between a heavy particle and the field of the pair leads, in fact, to an r^{-3} interaction law, rather than r^{-5}, and one can expect that a further integration would give rise to an even stronger dependency between ψ and φ particles, and would perhaps lead to an r^{-1} law. Therefore, one can consider the components of the ψ-φ pair in (14), (16), etc., to be either those of electron-positron, electron-neutrino, or neutrino-anti-neutrino. In his recent paper, A. Sokolow has examined the radiation of electron-positron and two neutrino pairs from an analogous standpoint. In fact, despite the fact that it was still a provisional investigation, it yielded the fact that one could arrive at the parallelism of two radiated neutrinos by an additional integration of (16), which is required precisely in order to fuse two "coherent" neutrinos into a photon; however, the photons lead to the Coulomb r^{-1} law.

If we compare formula (16), which one can also write down most conveniently in Fourier components, with the Dirac formula for pair-creation (15) then we remark that there is a certain *reciprocity* here. Namely, one can exhibit the vector potential A in the matrix element (15) as something that originates in the transition of heavy particles, and can then write:

$$\Box A = e\chi^{+}v\chi\,, \qquad A = \frac{e\chi^{+}v\chi}{\Box}\,. \tag{17}$$

In formula (16) (which only represents the coupling with the field, and not the matrix element of the radiation probability), however, we do not see the operator $\Box^{-1}$ applied to heavy particles, but to light ones. Conversely, one can replace the density of heavy particles in the Fermi formulas (14), (16), etc., with the associated potential. A closer examination of this "reciprocity," which is formally connected with the displacement of the $\Box$ operator, must, above all, yield the cases for which the "immediate" direct – or Fermi – probability for the radiation of pairs, on the one hand, and the usual – or Dirac – one, on the other, so the "reciprocity" becomes an identity. Since the constant g' and the choice of degree of coherence – i.e., the formulas (14), (16), or perhaps the K-U Ansatz – are still undetermined, one can adjust the immediate pair creation as perhaps less intensive than the Dirac expression for a few million volts, but more intensive for much larger energies.

The hypothesis that a proton can radiate a pair directly, instead of photons with $h\nu \geq 2mc^2$, leads to the possibility of explosive showers of electrons and positrons that are produced by protons, in precise analogy to Heisenberg's electron-neutrino showers. It seems reasonable to also consider an analogous "immediate" pair-radiation by an electron itself, which leads to a truly nonlinear wave equation for the electron (with an additional term of the form $g'\psi^3$), which allows the formation of nonlinear explosive showers multiplicatively, along with the quantum-electrodynamical ones ([1]).

However, we would like to leave this question, which is apparently closely linked with the more precise description of the two heavy particles (whether an anti-proton or a neutron plays that role), to a later discussion.

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([1]) Cf., our earlier remark: D. Iwanenko and A. Sokolow, Verh. Sibir. Phys.-techn. Inst. **4** (1936), 70. (Russian)