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Circulation relative to the Earth

By **V. Bjerknes**

Translated by D. H. Delphenich

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1. – In some previous works ([1]), I have referred to the advantage that one gains when one bases one's discussion of the motions of air and the oceans upon Lord Kelvin's concept of the circulation of closed curves. Therefore, for the sake of simplicity, I concerned myself with only absolute motion, and likewise neglected friction. In that way, one gains the advantage of being able to study the *primary* causes of motion in their purest forms. However, when one no longer discusses purely-theoretical questions, but concrete practical applications, considering the rotation of the Earth and friction will become unavoidable. The first oversight is very easy to correct, and that is what will be done in what follows. Things are different with friction. In general, there is no mathematical difficulty in writing down the line integral the represents the influence of friction on the circulation of a curve. However, if one starts from the rational theory of friction then that integral will depend upon the infinitely-small differences in velocity, and those are quantities that are not accessible to *observation*. That is why a direct application of that integral is not possible in practice, such that one must necessarily go down an indirect path in order to learn about the influence of friction more precisely. However, in order to enter that indirect path, one must represent the influence of friction in the equations by a symbol. Therefore, it will also be introduced in what follows, and indeed all the more so because the explicit appearance of that symbol is useful for the purely-qualitative discussion. That is because in such a discussion, one mostly comes to recognize the direction in which the influence of friction acts, and that is usually easy to do.

2. The kinematical relation between the absolute and relative circulation. – Lord Kelvin's concept of circulation is a measure of the motion of a curve that closes back into itself. When U_a is the velocity of an arbitrary particle on the curve, referred to a coordinate system, and U_a^t is the projection of that velocity onto the tangent of the curve, the circulation C_a of the curve will be defined by the integral:

$$C_a = \int U_a^t \, ds \,, \tag{1}$$

([1]) **V. Bjerknes**. "Ueber einen hydrodynamischen Fundamentalsatz und seine Anwendung besonders auf die Mechanik der Athmosphäre und des Weltmeeres," Kongl. Svenska Vetenskapsakademiens Handlingar **31** (1898). "Das dynamische Princip der Cirkulationsbewegungen in der Atmosphäre," Meteorol. Zeit. (1900).

in which ds is a line element along the curve, and the summation extends over the entire closed curve.

Furthermore, U means the velocity of a point on the curve, relative to the rotating Earth, and U^t is the projection of that velocity onto the tangent to the curve. From the principles of relative motion, one then has:

$$U_a^t = U^t + U_0^t,$$

in which U_0^t is the tangential component of the velocity U_0 of a point that instantaneously coincides with the material point in question along the curve, but is rigidly coupled with the Earth, and therefore moves with the motion of the Earth. Upon substituting that in (1), the absolute circulation C_a of the curve will split into two partial circulations:

(2) $$C_a = C + C_0 ,$$

in which C is the circulation of the curve relative to the rotating Earth:

(3) $$C = \int U^t ds ,$$

and in which C_0 represents the circulation of a curve that instantaneously coincides with the curve in question, but whose points are all rigidly coupled with the Earth:

(4) $$C_0 = \int U_0^t \, ds .$$

The latter circulation along a curve that is rigidly coupled with the Earth is easy to calculate.

We can first consider the special case in which the curve has the form of a parallel circle, which is especially important in the applications to the large atmospheric circulation. When such a curve is rigid and rigidly coupled with the Earth, all of its points will have one and the same velocity ωr, in which ω is the angular velocity of the Earth, and r is the radius of the parallel circle. Upon substituting that in (4), ωr will move outside of the integral sign. Only ds will remain inside of it, and the integral of the line element will give the length $2\pi r$ of the parallel circle. The value of C will then be $2\omega \cdot \pi r^2$, or:

(5) $$C_0 = 2\ \omega S ,$$

in which S represents the area of the parallel circle.

More generally, we can consider an arbitrary curve that is contained in the plane of a parallel circle. The velocity of a point on the curve that has a distance r from the Earth's axis will again be ωr. If θ represents the angle between that velocity and the tangent to the curve then we will find that the tangential component of the velocity is $U_0^t = \omega r \cos \theta$. If we substitute that in (4) and take the constant factor 2ω outside of the integral sign then we will have:

(6) $$C_e = 2\omega \int \tfrac{1}{2} r \cdot \cos\theta \, ds \,.$$

However, as one easily sees the quantity $\frac{1}{2} r \cdot \cos\theta \, ds$ is the area of the elementary triangle whose sides are the line element ds and the two vector radii to the endpoints of that line element. Therefore, the integral represents the area S that the closed curve bounds, and C_0 again reduces to the simple form (5).

Finally, one easily sees that an arbitrary rigid curve that is rigidly coupled with the Earth has the same circulation as its projection onto the equatorial plane or onto the plane of an arbitrary parallel circle. That is because a point on the given curve has the same velocity as its projection onto the equatorial plane, namely, ωr, when r is the radius vector to the projected point. If ds represents the line element of projected curve, and θ is the angle between the velocity ωr and that element then $\omega r \cos \theta \, ds$ will be the quantity that is to be integrated along the projected curve. Now, the element ds corresponds to an element of length $ds / \cos \psi$ on the given curve, where ψ is the angle that the given line element makes with the projected one. On the other hand, the velocity ωr has a projection onto the tangent to the given curve of $\omega r \cos \theta \cos \psi$, and the quantity to be integrated will then be once more $\omega r \cos \theta \, ds$, such that the two integrals will be identical to each other. We also return to formula (6), and from there, to formula (5) in the most-general case then, such that in all cases, the quantity S can be defined to be the area of the surface that is bounded by the projection of the curve onto the equatorial plane. That will then give the following result, which is the basis for the transition from the consideration of absolute circulation to the consideration of relative circulation:

I. – *The circulation of a rigid curve that is rigidly coupled with the Earth is equal to the area of its projection onto the equatorial plane, multiplied by twice the angular velocity of the Earth.*

It should be emphasized that this area is given in the form of a quantity with a sign by the integration (6), which agrees with the known principle for the sign of surface area, which says that one must link it with a chosen sense of traversal on the bounding curve.

From that result, formula (2) will go to:

(7) $$C_0 = C + 2\omega S \,,$$

which is the desired relation between the absolute and relative circulation of a curve.

3. The dynamics of relative circulation. – When the individual point on a material curve is acted upon by a driving force, its absolute circulation will change in time according to an equation of the form:

(8) $$\frac{dC_a}{dt} = F.$$

F is the line integral of the tangential component of the acceleration forces that act upon the individual point on the curve then and can be briefly referred to as the *circulation-creating force*. In the calculation of that force, one can always ignore accelerating forces whose integrals along the closed curve are equal to zero. Ultimately, the great utility of the concept of circulation lies in the simple form of equation (8). One comes to that simple form since, as **Lord Kelvin** has shown, the line integral of the acceleration of the individual point on a closed curve is equal to the derivative of the circulation of the curve with respect to time.

In order to go over to the corresponding equation for the case of the relative motion, we substitute the value of C_0 in (7), which will give:

$$\frac{d\,(C+2\,\omega S)}{dt} = F$$

or

$$\frac{dC}{dt} = F - 2\,\omega \frac{dS}{dt}\,. \tag{9}$$

The term $-\,2\,\omega \dfrac{dS}{dt}$ appears in the last equation in the same way that a circulation-creating force would, just like F. That apparent circulation-creating force depends, on the one hand, on the angular velocity ω of the Earth, and on the other hand, on the rate $dS\,/\,dt$ with which the area of the projection of the curve onto the equatorial plane changes. We will call that rate, when endowed with the negative sign, the *rate of contraction* of the area. A comparison of equations (8) and (9) will then give the following result:

II. – *The relative circulation of a material curve can be treated as if it were an absolute one, assuming that one extends the circulation-creating force that actually occurs by a fictitious force whose magnitude one will find by multiplying twice the angular velocity of the Earth times the rate of contraction of the area that is bounded by the projection of the curve onto the equatorial plane.*

That theorem corresponds completely to **Coriolis'** theorem on the relative motion of a single material point and can also obviously be derived from that theorem. However, the autonomous derivative above is decidedly the simplest.

4. The most-general hydrodynamical theorem on absolute circulation. – Now, the material curve initially belongs to a frictionless fluid or gaseous body, and no external forces besides gravity shall act upon it. Gravity does not act in such a way as to create circulation, since its acceleration will have a line integral along any closed curve that is equal to zero. The circulation-creating force F then reduces to the line integral of the accelerating force that originates in the gradient. The absolute circulation of each of the material curves that belong to the fluid will then obey the equation:

(10)
$$\frac{dC_a}{dt} = A,$$

in which A is given by the integral:

(11)
$$A = -\int v\, dp\,.$$

Here, v means the specific volume, and p means the pressure in the fluid. The fact that A represents the line integral of the acceleration force that originates in the gradient is explained immediately when we write dp in the form $(dp\ /\ ds)\ ds$ and recall that $-\ dp\ /\ ds$ represents the tangential component of the gradient.

For the practical calculation of the value of A along the curve where one makes one's observations, it is best for one to use the integral expression (11). However, when one deals with qualitative discussions, another interpretation of A is very convenient. Namely, by a conversion of the integral (11), one will see that one can define A to be a number that one arrives at as follows: One thinks of the isobaric surfaces as having a pressure difference of unity, and the isosteric surfaces (i.e., surfaces of equal specific volume) are drawn to have unit differences in the specific volume. Those two families of surfaces decompose all of space into a system of tubes, namely, the unit isobaric-isosteric tubes, and A will be number of such tubes that lie inside of the closed curve ([1]).

Before we go on to the case of relative motion, we will first generalize equation (10) by adding a term $-\ R$ that should represent the influence of the frictional resistance on the circulation of the curves. (10) will then become:

(12)
$$\frac{dC}{dt} = A - R\,.$$

There would be no difficulty in writing out an integral from which one can calculate R, formally speaking, that corresponds completely to the integral (11) for the calculation of A ([2]). However,

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([1]) Obviously, that will first become *exactly* true when one chooses infinitely-small units for the pressure and the specific volume. I have called the corresponding infinitely-thin unit tubes *solenoids*. The name might seem a bit strange, and for many readers, it might suggest Ampère's electrodynamical representations. However, the terminology is in complete agreement with the terminology of modern vector analysis, although the connection to the form of the theorem that was given here does not emerge clearly. Lord **Kelvin**, in direct connection with Ampère's terminology, has introduced the expression "solenoidally-distributed magnetism," and from it, he developed the general expressions "solenoidally-distributed vector quantities" or "solenoidal vector quantities." One then intends that to mean vector quantities whose spatial distribution can be given with the help of a system of tubes. The solenoids that come into question here, whose walls consists of isobaric and isosteric surfaces are now, in fact, the vector tubes of a solenoidal vector quantity, namely, the "vorticity gradient," to which can reduce the study of the formation of circulatory motions and vortices. One will be led to the consideration of those vector quantities when one formulates the theorems in question as theorems about vorticity, following **Helmholtz**, instead of as theorems on circulations, as Lord **Kelvin** did. (Confer my treatise "Ueber die Bildung von Cirkulationsbewegungen und Wirbeln," Videnskabsselskabets Skrifter, Christiania, 1898.)

([2]) When one lets κ denote the coefficient of friction and further lets U denote the velocity, as above, while v denotes the specific volume, one will find that when one starts from the equations of motion of viscous fluids and neglects some things that are allowable in atmospheric motions, one will have:

whereas the integral (11) is suited to direct practical applications since ordinary meteorological observations give satisfactory data for one to perform the calculations, so one can never make a corresponding use of the integral of R since the infinitely-small velocity differences that appear in it are not accessible to observation. Rather, that is why one must go down the opposite path and seek to determine R from equation (12) or the corresponding equation (13) below for the relative motion in those cases where all of the remaining terms in the equation are known from observations. If one recalls that fact then one can understand the meaning of the explicit appearance of the term R in the equation. Similarly, it is useful to recall the quantitative discussions in which it is not so much the magnitude of the frictional resistance that matters, as its direction. The direction or sign that R should have in equation (12) must be found by discussion in each case.

6. Circulation relative to the Earth. – With the help of theorem (11), we now go from equation (12) to the following one:

$$\frac{dC}{dt} = A - R - 2\omega\frac{dS}{dt} \tag{13}$$

with no further analysis. That equation will then be true for any curve that returns to itself and consists of particles in the atmosphere or ocean, and it describes the circulation of that curve as we would see it on the rotating Earth. We start from the fact that the only forces that affect the motion of particles in the air or ocean in a noticeable way are:

1. The force of gravity, which includes the centrifugal force that is due to the rotation of the Earth.

2. The gradient of the pressure.

3. Friction.

4. The "deflecting force of the Earth rotation."

We can then assert that all circulations in the air or ocean must obey equation (13).

Any problem in theoretical mechanics is one of prognosis when it is posed in its direct form, just as most of the known problems in practical meteorology are. The goal is to predict the dynamical physical state of the atmosphere at a later time when that state is known with sufficient precision at a given time. On the other hand, the problem can also appear in the opposite formulation: When the changes in the atmospheric state are known during a certain time interval,

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$$R = \int \kappa\, \nu\, (\mathrm{curl}^2\, U)_t\, ds,$$

in which curl is the symbol of a known operation in vector analysis, and the subscript t means that only the component of the vector $\mathrm{curl}^2\, U$ comes into question, as usual. This integral shows its true significance by guiding the search for the indirect path that might guide the determination of R. However, we will not take up that problem here.

deduce the physical and dynamical states that are the causes of the changes that occurred. It will be one part of that prognostic problem or its opposite that solves equation (13). When applying it, one must observe the following, above all:

1) The equation is true for any closed curve that consists of air particles. One is then completely free to choose the form and configuration the curve might have at the initial time. Therein lies one of the most important advantages of the method. One can make that choice on the basis of exclusively the convenience with which one can utilize the observational data or simplify the calculations. Obviously, the position that the curve has relative to the solenoids at that initial time plays the most important role. Now, the solenoids always run very close to parallel with the Earth's surface. For that reason, curves that run at a constant height above sea level will not enclose solenoids, practically speaking. On the other hand, curves that are included in vertical planes at the initial time, and which then consist of two horizontal curve segments that lie above each other and are linked with each other by two vertical curve segments, will enclose the largest number of solenoids that might enclose a curve under otherwise-equal circumstances. One's considerations can mostly be restricted to curves from those two types. We shall refer to them briefly as the *horizontal* and *vertical* ones with no concern for the fact that they will mostly not remain horizontal (vertical, resp.) in the course of their later motion. In connection with that, it deserves to be emphasized that due to the predominately horizontal direction of the wind, the particles of a curve will always remain for a relatively-long time at the level that they once assumed. It will then follow that a horizontal curve will always remains horizontal for a relatively-long time, and that a vertical curve will enclose one and the same bundle of solenoids for a relatively-long time, even if it does not also remain vertical.

2) If the form of the curve has been chosen then the first thing that one is required to know is *the motion of the curve at the initial time*. That is inferred from observations of the wind, and from measurement of cloud motions when one is dealing with curves or curve segments in the higher air layers. If one knows the velocity at a sufficient number of points on the curve from those observations then one calculates the circulation C from the tangential components and the rate of contraction $-dS / dt$ of the projection of the area onto the equatorial plane from the normals components. In that way, the last term on the right in equation (13) will be known, and likewise the initial value of C that is to be used in the integration.

3) When observations of *pressure*, *temperature*, and *humidity* have been made at a sufficient number of points that are distributed along the curve, one can calculate the specific volume v from that then ascertain the number A of solenoids inside the curve with the help of the integral (11).

4) The third term R that occurs on the right-hand side of the equation can never be found by direct observations, as we have stressed before. Rather, one must seek to calculate R from the equation itself in cases where all of the remaining terms are known with sufficient precision. Naturally, that is suited, above all, to the case of stationary circulation, where C is independent of time, such that the equation will reduce to:

$$A - R - 2\omega\,\frac{dS}{dt} = 0\,. \tag{14}$$

If one has learned the value of R in certain cases in that way then one can employ that value in the general equation (13) in analogous cases.

We shall not go into the applications of the theorem in detail here. I shall communicate only the most-relevant consequences that I likewise communicated before in my lectures at the Stockholm Institute in the Spring semester of 1901, along with the theorem itself, and I shall also add a few general remarks. My audience at the time was **Sandström**, who undertook the task of working out the further consequences, and I shall permit myself to refer to his publications on that subject.

7. Onset of motion from a state of relative rest. – If rest prevails relative to the Earth then the area S of the projection of a curve onto the equatorial plane will not change, and the last term in equation (13) will vanish. The term R, which depends upon the differences in velocity, will be likewise zero, and the equation will reduce to:

$$\frac{dC}{dt} = A\,. \tag{15}$$

According to that equation, the relative circulation will then begin from a state of relative rest. The equation has precisely the same form as the one that is valid for the absolute circulation in a frictionless fluid, which is equation (10). When one deals with the start of the motion, one can ignore friction as well as the deflecting force of the Earth's rotation, and we will have nothing to add to what was communicated in the previous articles as long as we are dealing with the *primary* causes of atmospheric motions.

The first cause of circulation then contained to be the appearance of solenoids, i.e., from temperature differences, and the first effect of those solenoids will be *a circulation of the curves that are contained in the vertical plane*, while the horizontal curves, which do not enclose any solenoids, will not contribute to circulation.

Now, if the Earth were at rest and the air were frictionless then the motion would progress as a circulation only in the vertical planes. However, as soon as the individual points of the vertical curves are set into motion, the terms $-R$ and $-2\omega\,\frac{dS}{dt}$ will also begin to contribute. In that way, it should be remarked that the infinitely-small velocity differences upon which the friction depends will first achieve large values relatively later in such a way that one would need to calculate a time with only the term $-2\omega\,\frac{dS}{dt}$. That term will have no influence on the circulation of the vertical curves for the time being. That is because the motion of those curves back to themselves will have no changes in its projections onto the equatorial plane as a consequence. By comparison, things are different with *horizontal curves*. One of them that surrounds a heated or cooled location in an annular way will exhibit a motion of expansion or contraction as a consequence of the initial circulation in the vertical planes. The area of its projection onto the equatorial plane will then increase or decrease, and the curve will go into circulation, even though it does not enclose any solenoids. As long as we can ignore friction, that will proceed according to the equation:

$$\frac{dC}{dt} = -\ 2\omega \frac{dS}{dt} .$$

That can be integrated for the time interval from t_0 to t during which the curve will neither enclose solenoids nor be retarded by friction:

$$C = -\ 2\omega\,(S - S_0)\ . \tag{16}$$

In that way, S_0 means the area of the projection at time t_0 . As long as the curve remains horizontal and the friction (*Reibung*) can be neglected in comparison to the viscosity (*dem trägen Widerstand*), the circulation of the curve will then increase in proportional to the decrease in the area of its projection onto the equatorial plane. When the friction comes into effect later, while the curve still remains horizontal, one will have to apply the more general equation:

$$\frac{dC}{dt} = -\,R -\ 2\omega \frac{dS}{dt} , \tag{17}$$

and when the circulation of the curve becomes stationary, one will have:

$$R = -\ 2\omega \frac{dS}{dt} , \tag{18}$$

which is an equation that is excellently suited to the determination of the value of R.

It is easy to see which direction those circulations will impart to the horizontal curves. Along such a curve, we can choose the direction of traversal to be the one that *follows* the rotation of the Earth or cyclonic (so S-E-N-W on the northern hemisphere) to be positive and the opposite or anticyclonic direction to be negative. That definition will become completely unambiguous when we avoid the unnecessary complication that a curve might intersect itself, and likewise overlook the case in which part of a curve lies on the northern hemisphere and part of it lies on the southern hemisphere. With that choice of positive direction of traversal, one finds by the integration (4) that the areas S will be a positive quantity, and that implies the simple rule:

III. – *A horizontal curve that contracts will take on a cyclonic circulation, while an expanding one will take on an anticyclonic circulation.*

From that theorem on the circulation of horizontal curves, we will once more find the known result that one otherwise derives from considering the horizontal components of the deflecting force of the Earth’s rotation. Therefore, to mention the most important cases: A curve that surrounds the entire Earth like a parallel circle and belongs to an upper air mass that flows towards the pole will contract, and that is why it will take on a circulation *with* the rotation of the Earth, so from W to E. By contrast, a similar curve that belongs to a lower air mass that flows to the equator will expand, and in that way take on a circulation that is against the rotation of the Earth, so from E to W. Moreover, a curve that surrounds the center of a cyclone down on the Earth in an annular way will assume cyclonic circulation, while one that surrounds the center of an anticyclone in the same way will assume an anticyclonic circulation.

7. [*sic*]. The reaction of horizontal circulations on the vertical ones. – Up to this point, the theory of circulation has led to no new qualitative results since it has only given us a tool for studying motions that have been known and understood for a long time with more quantitative precision. Things will change when we take the next step.

Once the circulation in the horizontal plane has begun, the originally-vertical curves, which were primarily brought into circulation by solenoids, will no longer remain vertical. That is because, on the one hand, the circulations of the horizontal curves in the initial phase down on the Earth's surface were mostly in the opposite direction to the ones in the upper air layers. On the other hand, even when the horizontal curves that lie on top of each other have circulations in the same directions, the motion of the upper curves would be retarded much more by friction. For that reason, in general, the upper and lower parts of the vertical curves will either proceed in different directions or also in the same directions, but with differing velocities. It follows from this that the original vertical curves can no longer remain vertical, as was said before, and that the areas S of their projections onto the equatorial plane will begin to vary. The vertical circulations will no longer depend upon the solenoids then, but also upon the horizontal motions. Therefore, as soon as sufficiently-intense horizontal motions arise, one must also apply the complete equation (13) for the discussion of the vertical circulations. Nothing would then prevent the last term, which depends upon the rotation of the Earth, from even taking the upper hand, such that circulations counter to the solenoids would be created.

In order to determine the direction of that vertical circulation that is independent of the solenoids, one can apply this general rule, which is useful even in the most complicated cases: Choose a certain direction of traversal on the given curve to the positive. That will give a well-defined direction of traversal for the projected curve in the equatorial plane to be positive, and the sign of the bounded surface will again be determined from that. If the area decreases algebraically then the given curve will take on circulation in the positive direction. However, that sequence of inferences can be shortened when one introduces the restrictions that the curve must lie on only one hemisphere and its projection onto the Earth or the equatorial plane must never intersect itself. In the initial position, where the curve is vertical, it will be projected onto the Earth's surface like a double line. However, a moment later, when it has assumed its inclined position, that double line will be converted into a closed curve. If one now follows through the sequence of inferences that was given with an arbitrary curve of that type and then compares the projections of everything onto the equatorial plane with the projections onto the surface of the Earth then one will arrive at the following rule:

IV. – *A curve that is originally vertical and then goes into an inclined position will be acted upon by a circulation-creating force whose direction can be found from the fact that it will be anticyclonic on the closed contour that defines the projection of the inclined curve into the surface of the Earth.*

The study of those vertical circulations that can take place independently of, or even against, the solenoids should be one of the most important applications of the theory of circulation. That is because that domain has not been explored to any great extent. However, the fact that one finds the solution to a known riddle there will be shown in the article by **Sandström** that follows next. In conclusion, I shall give only a first application, namely, to the case in which the relationships are simple enough that one has already found the elementary qualitative explanation some time ago.

8. James Thomson's theory of the grand atmospheric circulation. – On an Earth at rest, the trade wind would be a purely north-south wind that would have to blow from the pole to the equator. However, in reality, it blows as a NE wind that is limited to a zone between the equator and the horse latitudes. By contrast, between the horse latitudes and the poles, one has predominately southwesterly winds. **Maury** (1855), **Ferrel** (1856), **James Thomson** (1857), and **Ferrel**, on a second occasion (1860 and 1889), have presented models for the connection between those SW winds and the large atmospheric circulation ([1]). The only one that does not include anything that it kinematically or dynamically impossible is that of **James Thomson** and the last one by **Ferrel**, which agrees with Thomson's completely. According to that model, the grand circulation at high altitudes goes unperturbed between the pole and the equator. However, between the horse latitudes and the equator, the lowest air layers would have a circulation that is opposite to the one that would be produced by the temperature distribution. However, **Thomson** already gave a completely-applicable qualitative explanation for that behavior, which seems paradoxical on first glance, in his first article: Due to the fast circulation from W to E at the higher latitudes, the air masses have greater centrifugal forces than at the lower-lying parts of the rotating Earth. Thus, a barometric minimum will be created over the polar region. However, the lowest air masses, which will be retarded by friction against the Earth, do not have enough centrifugal force to resist the gradient of that minimum. That lowest air layer must then flow towards the pole.

Now, the theory of circulation will lead to exactly the same result, and indeed in such a form that one can subject the qualitative explanation to a quantitative test. We assume (as in **Hann**'s climatology) that a temperature difference of 34° exists between the mid-years for the parallels 35° and 80°, and since we know nothing more exact about the decrease in temperature with height, we assume, for simplicity, that it is equally large at both parallels. Those data will suffice to estimate the number A of solenoids between those two latitudes with the help of the integral (11). In the layers that lie above each other every 1000 m in altitude, one finds, say, 13×10^6 cm^2 / s solenoids, on average, and when those are the only things that generate circulation, they would create a

([1]) One finds all of the models together in **J. Thomson**'s treatise "On the grand Current of atmospheric Circulation," Trans. Roy. Soc. London (1892).

motion that would proceed towards the equator down at the Earth's surface and towards the pole in the higher air layers. However, the air masses generally move out of the W, and indeed they move faster in the upper layers than they do in the lower ones. A curve that consists of air particles and is contained in a meridian plane at the initial time will then assume an inclined position at a later moment in time since the upper part will be moved further towards the E than the lower one. It will now project onto the Earth like a closed curve, and the anticyclonic direction of traversal on that projected curve will determine a direction of circulation on the original vertical curve that points towards the pole down at the surface of the Earth and towards the equator higher up, as **Thomson** found by considering the centrifugal force. Now, one can easily calculate the value of dS / dt for which the two causes of circulation will be in equilibrium. For one curve that extends form 35° to 80° and whose upper branch lies 1000 m higher than its lower one, one must then have $2\omega \frac{dS}{dt} = 13 \times 10^6$. One finds that this condition will be fulfilled when W-wind increases by 2.2 m/s, on average, for every 1000 m in altitude. Now, the strength of the wind is known to first increase rapidly with altitude and them more slowly. As long as the W-wind increases more than 2.2 m/s per 1000 m of altitude, the circulation must them go towards the pole lower down and towards the equator higher up. However, as soon as an altitude is reached at which the increase in the W-wind is smaller, the circulation will have the direction that would expect from the temperature distribution.

Those numbers are cited here only as an example of how one can utilize the theory of circulation for quantitative investigations. However, they can prove to be correct, up to order of magnitude, and the path that one has to follow when a sufficient amount of observational data has been assembled is entirely clear. For the implementation of such calculations in practice, I shall refer to the article by **Sandström** that follows (in the next issue).