"Über Friedrich Kottlers Abhandlung 'Über Einsteins Äquivalenzhypothese und die Gravitation'," Ann. Phys. (Leipzig) **51** (1916), 639-642.

On Friedrich Kottler's treatise "Über Einsteins Äquivalenzhypothese und die Gravitation" ([1])

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Among the works that have addressed the general theory of relativity critically, those of **Kottler** are especially noteworthy, because that expert has actually penetrated into the spirit of the theory. I would like to analyze the last of those articles here.

Kottler asserted that in my later work I had abandoned the "equivalence principle" that I had proposed, by which I sought to unify the concepts of "inertial mass" and "gravitational mass" into a single concept. That opinion must be based upon the fact that we are not both referring to the same "equivalence principle," because in way of looking at things, my theory rests upon that principle exclusively. For that reason, let me reiterate the following facts:

1. **The special theory of relativity as a limiting case**. – Let a finite region of space-time be free of any gravitational field, i.e., let it be possible to construct a reference system K (i.e., a "Galilean system"), relative to which the following things are true for that region: The coordinates are directly measurable in the known way by a unit yardstick, and time is directly measurable by a unit clock, as one cares to assume in the special theory of relativity. An isolated material point will move uniformly and rectilinearly relative to that system, just as **Galileo** had assumed.

2. **The equivalence principle**. – Starting from that limiting case of the special theory of relativity, one can ask whether an observer in the region considered that is uniformly accelerated relative to K must perceive his state to be accelerated or whether he can maintain the perception that his state can be interpreted as one of "rest" according to the (approximate) known laws of nature. More precisely: Do the known laws of nature (to a certain approximation) allow us to consider a reference system K' that is uniformly accelerated with respect to K to be a system at rest? Somewhat more generally: Can the principle of relativity also be extended to reference systems that are (uniformly) accelerated relative to each other? The answer is: To the extent that we actually know the laws of nature, nothing prevents us from considering the system K to be at

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rest when we assume that a gravity field (which is homogeneous in the first approximation) is present relative to K', because all bodies will fall with the same acceleration independently of their physical nature relative to our system K', just as they do in a homogeneous gravity field. I call the assumption that one can treat K as being at rest with full rigor the "equivalence principle," and without it, no natural law would be fulfilled relative to K'.

3. **The field of gravity is not just required kinematically**. – One can also invert the argument above. Let the system K' that was formed in the field of gravity that was considered above be the original one. One can then then introduce a new reference system K that is accelerated with respect to K' and relative to which (isolated) masses (in the region in question) will move uniformly and rectilinearly. However, one may not proceed further and say: If K' is a reference system is provided with an *arbitrary* gravitational field then a reference system K can always be found relative to which isolated masses will move uniformly and rectilinearly, i.e., relative to which no gravitational field will exist. The absurdity of such an assumption is clear. If the gravitational field relative to K' is, for example, that of a mass-point at rest then that field certainly cannot be transformed away in the entire neighborhood of the mass-point by any trick of transformation, no matter how clever. One can then by no means assert that the gravitational field can be explained in a purely kinematical way in any sense of the word. A kinematical, non-dynamical, conception of gravitation is not possible. We do not then come to understand *arbitrary* gravitational fields merely by means of transformations from one Galilean system into another by acceleration transformations then, but by means of ones of an entirely special kind that must still satisfy the same laws as all other gravitational fields. That is just another way of formulating the equivalence principle (especially in its application to gravitation).

A theory of gravitation will then abandon the equivalence principle, in the sense that I understand it, only when the equations of gravitation are *not* fulfilled in *any* reference system K' that moves non-uniformly relative to a Galilean reference system. The fact that this reproach cannot be made against my theory with its *generally*-covariant equations is obvious because those equations are fulfilled relative to any reference system then. *The demand of the general covariance of the equations subsumes that of the equivalence principle as an entirely special case.*

4. **Are the forces of the gravitational field "real" forces?** – **Kottler** reprimanded me that I had interpreted the second term in the equations of motion:

$$\frac{d^2 x_\nu}{ds^2} + \sum_{\alpha,\beta} \begin{Bmatrix} \alpha\ \beta \\ \nu \end{Bmatrix} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds} = 0$$

as expressing the influence of the field of gravity on the mass-point, while the first term expressed the Galilean inertia, to some extent. In that way, "actual forces of the field of gravity" were introduced, which did not correspond to the spirit of the equivalence principle. In response to that, I reply that the equation, as a whole, is generally covariant, so it is consistent with the equivalence principle in any event. The terminology that I introduced for the terms is largely meaningless and solely intended to accommodate our way of thinking about physics. That is also especially true for the concepts:

$$\Gamma_{\alpha\beta}^{\nu} = -\begin{Bmatrix} \alpha\ \beta \\ \nu \end{Bmatrix}$$

(viz., the components of the gravitational field) and t_{α}^{ν} (viz., the energy components of the gravitational field). The introduction of that terminology is largely unnecessary, but it seems to me that is not worthless for the sake of maintaining continuity of thought, at least for the time being. For that reason, I have introduced those quantities, despite the fact that they do not possess a tensor character. However, the equivalence principle is always satisfied when the equations are covariant.

5. – It is true that I had to purchase the general covariance of the equations at the expense of giving up the usual measurement of time and the Euclidian measurement of space. **Kottler** believed that he could get along without that sacrifice. However, even in the case that he considered of a system K' that is accelerated relative to a Galilean system in the **Born** sense, one must abandon the usual measurement of time, since from the standpoint of the theory of relativity, it is already obvious that one must also give up the usual measurement of space. **Kottler** himself was certainly convinced of that necessity when he sought to undertake the theoretical plans that he anticipated.

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