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Application of Lenz's principle to the phenomena that accompany the charging of condensers

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I shall first summarize the proofs that Pellat ([1]) and Sacerdote ([2]) gave in regard to those phenomena in just one proof by using a remark that was made by Massieu.

Suppose that a *closed condenser* has thin armatures that are glued to the dielectric. The external armature B is connected to the case, while the internal one raised to the potential $V > 0$.

The state of that condenser depends upon V, its temperature T, and the uniform pressures P and p, one of which prevails externally, while the other one prevails internally.

Suppose that P is constant.

Under the influence of the variations dV, dT, dp, the charge M on A will increase by dM, the dielectric will receive a quantity of heat dQ, the internal volume (of the cavity) v will increase by dv, and the external volume v' will increase by dv'.

The increase in energy in the condenser under that transformation will be:

$$dU = V\,dM + J\,dQ + p\,dv - P\,dv'. \tag{1}$$

If that transformation takes place in a reversible manner then dU, $d\,(MV)$, $d\,(pv)$, and $d\,(Pv')$ will be exact differentials, and the same thing will be true for:

$$dX = dU - d\,(MV) - d\,(pv) + d\,(Pv') = -\,M\,dV + J\,dQ - v\,dp\,. \tag{2}$$

Set:

$$dQ = a\,dV + b\,dp + c\,dT \tag{3}$$

([1]) PELLAT, J. de Phys. (3), t. VII, pp. 18.
([2]) P. SACERDOTE, *Thèse de doctorat*, Paris, 1899, pp. 9.

and write out that dX and dS are exact differentials:

(4) $$-\frac{\partial M}{\partial p}+J\frac{\partial a}{\partial p} = J\frac{\partial b}{\partial V}-\frac{\partial v}{\partial V},$$

(5) $$-\frac{\partial M}{\partial T}+J\frac{\partial a}{\partial T} = J\frac{\partial c}{\partial V},$$

(6) $$J\frac{\partial b}{\partial T}-\frac{\partial v}{\partial T} = J\frac{\partial c}{\partial p},$$

(7) $$\frac{\partial a}{\partial p} = \frac{\partial b}{\partial V},$$

(8) $$\frac{\partial a}{\partial T}-\frac{a}{T} = \frac{\partial c}{\partial V},$$

(9) $$\frac{\partial b}{\partial T}-\frac{b}{T} = \frac{\partial c}{\partial p}.$$

Hence, upon taking into account that $M = CV$:

(10) $$\frac{\partial v}{\partial V} = \frac{\partial M}{\partial p} = V\frac{\partial C}{\partial p},$$

(11) $$a = \frac{T}{J}\frac{\partial M}{\partial T} = \frac{T}{J}V\frac{\partial C}{\partial T},$$

(12) $$b = \frac{T}{J}\frac{\partial v}{\partial T}.$$

1. *Electrical dilatation of condensers.* – One easily infers from (10) that:

(13) $$\frac{\Delta v}{v} = 3\,(\alpha+\kappa)\,\tau,$$

in which:

α denotes the inverse of the Young modulus,

κ denotes the expansion coefficient of the dielectric constant under uniform traction that is perpendicular to the lines of force,

τ denotes the energy per unit volume in the dielectric.

2. *Adiabatic heating θ of the dielectric during charging at constant pressure.* – One easily deduces from (3) and (11) that:

$$\theta = -\frac{T}{J}\frac{h}{d\gamma}\tau, \tag{14}$$

in which h denotes the coefficient $\frac{1}{C}\frac{\partial C}{\partial T}$, d denotes the density, and γ is the specific heat of the dielectric at constant pressure and potential.

There is heating then if $h < 0$ (e.g., glass and paraffin) and cooling if $h > 0$ (e.g., ebonite). Those variations of temperature do not exceed 10^{-5} for glass and 10^{-4} for ebonite, moreover.

2. *Application of the generalized Lenz principle.* – When one increases the difference in potential between the two armatures of a condenser, not only will the capacity increase, which will oppose the variation of V, but each of the immediate factors of C (viz., K, S, and e) will vary in the sense that is consistent with producing that increase in the capacity. Indeed, it has been known for some time that S increases and e diminishes under the influence of electrostatic tension, and the experiments of Ercolini have removed all doubt about the increase in K. However, one could have assumed that the latter phenomenon was *necessary* by an extension of Lenz's principle.

By contrast, the increase in the capacity is, in turn, opposed by the variables upon which it depends other than V. Hence, the extension of S is opposed by elastic reactions that are developed. Similarly, if the condenser were hermetically sealed then the increase in the internal volume would provoke a reduction in pressure that would react against the extension of the walls.

Likewise, the *temperature of the dielectric varies in a sense that makes that variation tend to diminish the capacity* [see formula (14)].

One then sees that one must be careful not to overgeneralize the application of the Lenz's principle of reaction (or conservation) by saying that all of the factors that the capacity depend upon (directly or indirectly) will conspire to make it increase. *That is true only for the immediate parameters*, i.e., the ones that act upon the capacity directly, namely, K, S, e.