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On the equations of dynamics

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Translated by D. H. Delphenich

Let $x_1, x_2, \ldots, x_m$ be the parameters that determine the positions of the points of a material system S at an arbitrary instant t, and let $x_1', x_2', \ldots, x_m'$ be their derivatives with respect to that variable t. I suppose that the *vis viva* integral exists, and I shall let T denote one-half the sum of the *vis vivas*, while U denotes a force function and h denotes the energy constant.

Let ξ represent an auxiliary unknown and consider the following expression:

$$T - \frac{\xi'^2}{U}. \tag{1}$$

The function U and the coefficients of the quadratic form T include $x_1, x_2, \ldots, x_m$ arbitrarily, but they do not contain ξ.

I imagine that one regards:

$$T - \frac{\xi'^2}{U}$$

as being one-half the *vis viva* of a material system S_1 to which no forces are applied. The equations of motion for the latter will be:

$$\left\{ \begin{aligned} & \frac{d}{dt}\left(\frac{\partial T}{\partial x_i}\right) - \frac{\partial T}{\partial x_i'} - \frac{\xi'^2}{U^2}\frac{\partial U}{\partial x_i} = 0 \qquad (1 \leq i \leq m), \\ & \frac{d}{dt}\left(\frac{\xi'}{U}\right) = 0. \end{aligned} \right. \tag{2}$$

There is one obvious immediate integral:

$$\frac{\xi'}{U} = \text{a constant } C.$$

I shall give the value 1 to C, and I will then have a particular solution for which the following relations are satisfied:

(3) $$\frac{d}{dt}\left(\frac{\partial T}{\partial x_i}\right) - \frac{\partial T}{\partial x_i'} - \frac{\xi'^2}{U^2}\frac{\partial U}{\partial x_i} = 0 .$$

Now, those are the ones that define the motion of the system *S*.

Equations (2), which include them, do not depend upon the constant *h*, and the choice of it remains arbitrary.

One sees that if one studies the motion of a system of points that is subject to the action of forces that derive from a potential, *in general*, and notably the transformations that one can apply to the equations of motion, then it will be permissible to confine oneself to the case in which the forces vanish, i.e., to the problem of generalized geodesics.

The results obtained for the last problem extend to that case, and without determining the energy constant, to the case in which the forces exist and admit a potential, which appears to be more general.

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