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The principle of the conservation of electricity

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I. – Principle of the conservation of electricity

The quantity of matter and the quantity of energy are not the only magnitudes that remain invariant in the world; the quantity of electricity enjoys the same property. If one considers an arbitrary electrical phenomenon in its totality then one will observe that the distribution of electricity can change, but that the sum of the quantities of free electricity will never vary, i.e., if the electric charge submits to a positive variation at certain points then it will submit to a negative variation at some other points, and at the same time, *the algebraic sum of all of the simultaneous variations will always be equal to zero.* The sum of the quantities of free electricity will then be invariable since its total variation is always zero. That law, which we will call the *principle of the conservation of electricity*, extends to all of the phenomena that have been studied up to now. It results from experiments that I will recall later on, which are old and in a sense elementary. Its statement in ordinary language is then simply a summary of some known facts. In return, its translation into analytical language leads to some new consequences. I propose to make that translation in this article, i.e., to express the principle of the conservation of electricity as an equation and to show the way that it can be used in the study of certain phenomena and the prediction of new facts in various examples.

When thus written in the form of an equation, the principle of the conservation of electricity will have exactly the same importance in analysis as the principle that is called *the equivalence* or *conservation of energy.* Indeed, the principle of the equivalence, when extended to electricity by the work of Helmholtz, W. Thomson, Joule, and other physicists, begins by providing a condition equation, as one will see later on. Now, the principle of the conservation of electricity provides a second equation, which is distinct from the one that is given by the principle of equivalence and is compatible with it, moreover. The two principles have exactly the same utility. One can, to abbreviate, give them the names of the *first* and *second principle of electricity.* The second one serves to double the resources of analysis by increasing the number of equations at one’s disposal from one to two.

Before translating them into algebraic language, I will first recall the phenomena for which the principle of conservation of electricity has been verified.

II. – Experimental proofs.

Those phenomena are the division of a charge between two conductors, and the development of electricity by friction, influence, or by piles.

When electric charge is divided between two bodies, only the distribution will vary, while the total charge will remain constant. One will find a first proof of that upon referring to the classical experiment by which Coulomb proved that electrical repulsion is proportional to the charge. One recalls that Coulomb measured the repulsion that was produced by a ball that was fixed in his balance when he touched that fixed ball with another auxiliary ball of the same diameter that was isolated and not electrified, and he found that after separating them, the electrical repulsion was reduced to one-half of what it was before. That experiment can be interpreted in two ways: If one defines the electric charges by the repulsion that they produce then that experiment shows that the charge on the fixed ball is reduced by one-half, and since the two balls take on equal charges, by reason of symmetry, it will then follow that the charge on the fixed ball is reduced to one-half of what it was and the total charge had only been separated. With that interpretation, the experiment shows that there is conservation of electricity. One can likewise say that Coulomb's experiment shows the proportionality between the repulsion and the electric charge. One then *assumes* that the charge on the fixed ball was reduced by one-half, and consequently, one assumes ([1]) that there is a simple division of the original charge with conservation of the quantity of electricity. Therefore, in one of those interpretations, one *assumes* the principle of conservation of electricity, while in the other, one *proves* it.

Furthermore, in all of the experiments where one measures the charge or the capacity of a conductor, one effects the division of certain charges and assumes that there is conservation of electricity. The agreement that exists between the results of the measurements that were made constitutes a multiple and precise verification of the principle on which one relies.

Thus, when there is a division of electricity between two conductors, one of them will gain precisely what the other one loses. In other words, the algebraic sum of the simultaneous variations of the charges is equal to zero.

One knows that when there is electrification by friction, pressure, or cleavage, the two bodies that take part in those actions will acquire new charges after their separation. However, those charges are equal and have opposite senses. Their algebraic sum is zero.

The same thing is true for influence that is true for friction. Faraday carefully showed that the algebraic sum of the quantities of electricity that are produced by influence are always zero.

Finally, one knows that the two poles of an open pile produce quantities of electricity that are always equal and opposite in sign. Those quantities will be neutralized continuously when one closes the circuit.

([1]) I say that one assumes the principle of the conservation of electricity in this case and that the symmetry argument is not sufficient to establish that the charge on the fixed ball is reduced to one-half. Indeed, the symmetry argument implies only that the charges on the two balls must be equal to each other. and it is clear that those charges can be equal to each other without being one-half of the original charge. For example, each of them can be one-fourth of the original charge.

In summary, no matter what electrical phenomena that one considers, the algebraic sum of all the simultaneous variations of the charges is equal to zero. That is a quantitative fact that is provided by experiments ([1]).

When one appeals to the hypothesis, or rather, to the notation of two electric fluids, one can express the same fact by saying that the two fluids always appear and disappear in equivalent quantities. The principle of the conservation of electricity was stated in that form a long time ago. The image of two fluids undoubtedly has its utility, moreover. However, the use of figurative language here would be a somewhat superfluous detour. In order to express a quantitative fact as an equation, it suffices to write it down in algebraic symbols.

III. – Analytical expression for the principle by an integrability condition.

Let a system of bodies in which one produces an arbitrary electrical phenomenon be given. One can imagine dividing that system into three parts A, B, C. Let a, b, c be the variations in electric charge that take place in A, B, C, resp., during a certain time interval. The principle of the conservation of electricity demands that one must have:

$$a + b + c = 0 .$$

Suppose that A traverses a closed cycle, i.e., one carries out an arbitrary series of changes of state on A that finally bring it back to its original state identically. In that case, one will have $a = 0$ and consequently $b + c = 0$. The variations of charge that are experienced by B and C are then equal and opposite in sign. In other words, A has restored all of the electricity to C that it took from B. In yet another way of saying that, the algebraic sum of the quantities of electricity that are received by A is zero. If one calls the variation of the infinitely-small charge that is incurred by A when the state of A varies infinitely little dm then one must have:

$$\int dm = 0$$

([1]) There is only a secondary difference between the experiments that we just spoke of and the ones that show that matter is indestructible, and more generally, between the measurement of a quantity of electricity and an ordinary weight, and it is the following one: When one puts a body into a balance, the total weight of that body is found immediately, i.e., without one having to account for its form. That amounts to saying that the ponderable body is only a point with respect the distance to the attracting mass, which can be supposed to be concentrated at the center of the Earth. In the measurements that are made with the electric balance, the attractions always involve small distances. That makes the calculation of the reduction more or less complicated when one takes the distribution into account. If one arranges that an electric mass M is constant and very distant then one can make it play the role of the Earth. The measurement of the quantity of electricity will then take the same form as that of weight by the chemist, and the principle of the conservation of electricity will be stated as follows: *No matter what actions take place in a system, the total electrical attraction that is experiences due to the point M will remain invariant.*

for any closed cycle that is traversed by *A*. In order for that to be true, it is necessary and sufficient ([1]) that *dm* must be an exact differential. Let *x*, *y* be the two independent variables that the state of *A* depends upon at each instant. The expression for *dm* will then have the form:

$$dm = X\, dv + Y\, dy\ .$$

As one knows, the condition for that expression to be an exact differential is:

$$(\alpha) \qquad \frac{\partial X}{\partial y} = \frac{\partial Y}{\partial x}.$$

That equation expresses the principle of the conservation of electricity.

I would like to show some of the uses that one can make of that equation in some examples. The path that will be followed is always the same in each case. Each time, one must define the independent variables that one considers and introduce them into equation (α). One thus expresses the principle of the conservation of electricity. In addition, in order to complete the analysis, it will be convenient to appeal to the principle of equivalence and likewise express it as an equation. One will thus arrive at a system of two equations that are distinct and compatible, and all that will remain is to discuss them and interpret them in plain language.

IV. – Examples of applications. Dielectric strength of a gas. Contraction of a gas produced by an electric influence.

As a first example of an application, I shall take the phenomenon that was discovered by Boltzmann in 1875 ([2]). Boltzmann proposed to measure what one calls the *dielectric strength of gases.* To that end, he arranged two parallel metallic plates *A*, *T* on fixed posts that formed the two armatures of a condenser, which wall placed inside the bell of a pneumatic machine. *A* was isolated, while *T* was connected to ground. One begins to charge the condenser by briefly connecting the plate *A* to one pole of a pile whose other pole is connected to ground; one then isolates *A*. When one increases the pressure *p* of the gas that is inside the bell, one confirms that the quantity of free electricity in *A* will diminish. The isolation remains perfect and the plate *A* will remain immobile, but the capacity of the condenser will become greater. When one introduces the gas into the bell, which is supposed to be vacuous to begin with, everything happens as if the distance between the plates had become *D* times smaller. The gas will then enjoy the property that its presence will make the capacity of the condenser *D* times greater. *D* is then the dielectric strength of the gas when the pressure is *p*. Boltzmann confirmed that *D* varied from one gas to another and that *D* varied in proportion to the pressure *p* for the same gas.

Let *m* be the quantity of free electricity in *A*. *m* depends upon two independent variables, namely, the potential *x* that one raises the plate *A* to and the pressure *p* of the gas. One then has:

([1]) See Note A.
([2]) Poggendorff's Annalen **155** (1875), pp. 403; J. de Phys. t. **5**, pp. 23.

$$(1) \qquad dm = c\,dx + h\,dp\,,$$

in which dm is the quantity of electricity that is received by the plate A when x increases by dx and p increases by dp. c is the capacity of the condenser when the gas is kept at the pressure p, h is a coefficient that is positive, according to Boltzmann's experiment, because when p increases by dp, the capacity of the condenser will increase, and consequently, in order to keep x constant, one must increase m by a positive quantity $h\,dp$. In order for dm to be an exact differential, it is necessary that one must have:

$$(\alpha) \qquad \frac{\partial c}{\partial p} = \frac{\partial h}{\partial x}.$$

That equation expresses the principle of the conservation of electricity.

In order to complete our study of the Boltzmann phenomenon, we agree to add another equation to equation (α) that expresses the principle of equivalence, which we shall calculate. To that end, we must show that the Boltzmann phenomenon permits us to traverse the plate A with a closed cycle such that work is transformed into electrical energy, or conversely. We will then write down that the expenditure of work is equal to the variation in the electrical energy.

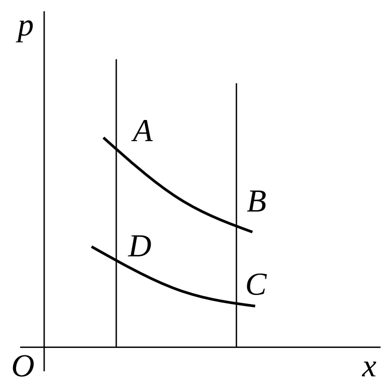

Figure 1.

Take the potential x and the pressure p to be the rectilinear coordinates of a point P (Fig. 1). Any variation of the state of the plate A will be represented by a line that is described by the point P. When one varies the pressure while keeping A connected to a reservoir at constant potential, the variation of A will be represented by a line that is parallel to the pressure axis, such as BC and AD. Those lines represent variations at constant potential. If one varies the pressure while leaving the plate A isolated then the potential x will vary at the same time. The variation in A will be represented in those cases by a line such as AB or CD. Those lines represent variation at constant charge. One can form a quadrilateral $ABCD$ out of two lines of the first type and two lines of the second type. That quadrilateral represents a closed cycle that can be traversed in one or the other sense. To fix ideas, suppose that the representative point that starts from C traverses the various points of the cycle in the order $CBADC$. The pressure increases from C to B, and consequently, the capacity increases, too. At the same time, the plate A is found to be connected to an electrical reservoir at a potential x_2, so electricity will be taken from that reservoir by A. The connection between A and the reservoir is interrupted at the point B. The pressure increases from B to A, and since the electric charge on A remains constant, its potential will diminish. At A, one connects the isolated plate to an electrical reservoir at the potential x_1 . The pressure diminishes from A to D. Consequently, the capacity will diminish, and electricity will be lost to the reservoir whose potential is x_1 . Finally, the plate A remains isolated from D to C, and it finally returns to its initial state at C. By definition, a certain quantity of electricity was taken to the reservoir whose potential is x_2 and transported into the reservoir whose potential x_1 . There will then be a reduction of the electrical energy that is contained in the system of those two reservoirs. On the other hand, in order to vary the pressure of the gas, one must displace a piston and consequently put a certain amount of mechanical work into a play. Since the cycle is closed, by virtue of the equivalence principle the mechanical work produced will be equal to the reduction in electrical energy. If one calls an infinitely-small variation of the volume D of the gas that is contained in the bell dv and the corresponding pressure p then $\int p\,dv$ will represent the work that is produced by the pressure that is exerted by the gas on the surface of the piston. On the other hand, if dm represents an infinitely-small quantity of electricity that is received by the plate A while that plate is connected with a reservoir whose potential is x then $\int x\,dm$ will represent the quantity of electrical energy that is lost. One will then have ([1]):

$$\int p\,dv = \int x\,dm ,$$

or, upon setting $p\,dv = x\,dm = d\mathcal{E}$:

$$\int d\mathcal{E} = 0$$

for a closed cycle. In other words, $d\mathcal{E}$ must be an exact differential. One can express the principle of the conservation of energy by writing out that this condition is fulfilled.

In order to express $d\mathcal{E}$ as a function of the independent variables x and p, one must express dv as a function of those variables. One then sets:

(2) $$dv = a\,dx + b\,dp ,$$

in which a is a coefficient that can be zero, because we shall once more ignore whether the volume of the gas can depend upon the potential x. v is a function of p, and possibly x. The expression for dv is then an exact differential. One will then have the relation:

(3) $$\frac{\partial a}{\partial p} = \frac{\partial b}{\partial x}$$

between the coefficients a and b.

Upon replacing dv with its value in the expression for $d\mathcal{E}$, one will get:

([1]) See Note B.

(4) $$d\mathcal{E} = (a\,p - c\,x)\,dx + (b\,p - h\,x)\,dp\,.$$

In order for $d\mathcal{E}$ to be an exact differential, one must have:

$$\frac{\partial(ap - cx)}{\partial p} = \frac{\partial(pb - hx)}{\partial x}\,,$$

or upon developing:

$$p\left(\frac{\partial a}{\partial p} - \frac{\partial b}{\partial x}\right) + a = x\left(\frac{\partial c}{\partial p} - \frac{\partial h}{\partial x}\right) - h\,.$$

The coefficient of p is zero by virtue of equation (3). One will then have simply:

(β) $$a = x\left(\frac{\partial c}{\partial p} - \frac{\partial h}{\partial x}\right) - h\,.$$

Equation (β) expresses the equivalence principle. Upon combining it with equation (α), one will see that equation (β) simplifies and that it will reduce to:

(β') $$a = -\,h\,.$$

The system of equations (a) and (b) is then equivalent to the system of equations:

(α) $$\frac{\partial c}{\partial p} = \frac{\partial h}{\partial x}\,,$$

(β') $$a = -\,h\,.$$

That is the result of our analysis.

We point out that those equations are distinct and compatible, since one of them contains a function a that is not contained in the other one. The two principles that it expresses are then distinct and compatible, in their own right, and they will have exactly the same importance as far as analysis is concerned.

One can now infer the physical laws that are contained implicitly in equations (α) and (β'). In the first place, we remark that the Boltzmann phenomenon can be expressed by saying that h is non-zero and positive. From equation (β'), it will then follow that a is non-zero and negative. Now, from equation (2), a is the partial derivative of v with respect to x. Thus, when x increases, v will decrease. Hence, the volume of a gaseous mass that surrounds the two armatures of an electric condenser will vary in the opposite sense to the potential that is acquired by that condenser. In other words, the electrification of a condenser will suffice to produce a contraction in the volume of the gas that forms the insulating layer. That is a new physical phenomenon that experiments have glimpsed, but not demonstrated. Analysis shows us its existence, and even permits us to calculate its numerical value. The equivalence principle, taken by itself, cannot lead to the result that a is necessarily non-zero. Indeed, if one does not take equation (α) into account then the equivalence principle will be expressed by equation (β), which can be satisfied even when a is zero. The principle of the conservation of electricity is then necessary if one is to conclude the phenomenon of the electrical contraction of gases from the Boltzmann phenomenon.

We propose to calculate the dilatation Δv that is experienced by the gas that is contained within the bell when one carries the plate A from the potential 0 to the potential x. Boltzmann found experimentally that the dielectric strength of a gas is proportional to its pressure. That dielectric strength is then equal to $1 + \gamma p$, where γ is a constant that is specific to the gas. If one calls the capacity of a condenser *in vacuo* c_0 then its capacity at pressure p will be $c = c_0 (1 + \gamma p)$. Upon substituting that value of c in the expression for dm and integrating, one will have:

$$m = c_0 (1 + \gamma p)\, x\,. \tag{5}$$

The integration constant is zero, because when the potential x is zero, the charge m will also be zero. We remark that from equation (5), the curves AB, CD in Fig. 1, which represent the relation that exists between p and x when the charge m is constant, are arcs of hyperbolas.

From equation (5), one has $h = c\,\gamma x$, and since from equation (β'), one has $\partial v / \partial x$ or $a = - h$, and it will follow that:

$$\frac{\partial v}{\partial x} = - c_0\, \gamma x\,,$$

and upon integrating:

$$\Delta v = -\tfrac{1}{2} c_0\, \gamma x^2. \tag{6}$$

That is then the law of dilatation. One sees that the variation of the volume is proportional to the constant γ that is specific to the gas employed, to the capacity c_0 of the condenser *in vacuo*, and to the square of the potential x to which the plate A has been carried ([1]).

We can calculate the numerical value of the dilatation that is experienced by the gas under certain well-defined conditions with the aid of equation (6). Suppose that the gas is air and that air forms the insulating layer in a condenser that is composed of two parallel metallic armatures. Let S be the surface of each armature, let e be the distance that separates them, and let v_0 be the volume of air that is enclosed by the cylindrical space that has the two armatures for its bases. Suppose

([1]) There seems to exist a very simple relationship between the electric dilatation of a gas and its optical properties that is worthy of mention: The constant γ is equal to the refracting power of the gas. Indeed, from Maxwell's theory, the dielectric strength of a body must be equal to the square of its index of refraction n. Boltzmann's experiments have precisely the goal of verifying that relationship, and indeed, it has been verified: The values of the dielectric strength or of $1 + \gamma p$ that were found by Boltzmann are reasonably equal to the squares of the indices of that gas; one then has:

$$1 + \gamma p = n^2,$$

so

$$\gamma = \frac{n^2 - 1}{p}\,.$$

that the air pressure is 760 mm of mercury and that one carries the potential x to the largest value that one can give it at that pressure, i.e., the value for which the explosive distance is equal to e. Nonetheless, it is clear that one cannot charge the condenser because a spark would fly above it between the two armatures. One seeks the ratio by which one can dilate the volume of air enclosed between the two plates and whose original volume is v_0 . One has $c_0 = S / 4\pi e$ and $v_0 = S e$, so $c_0 = v_0 / 4\pi e^2$. Upon substituting that value in equation (6), one will get:

$$\frac{\Delta v}{v_0} = -\frac{1}{8\pi}\gamma\frac{x^2}{e^2} .$$

The left-hand side represents the electrical dilatation per unit volume of air. In the right-hand side, if one supposes that x corresponds to the explosive distance e then the quotients x / e will be the value of the potential that gives an explosive distance that equals one unit of length. According to Sir W. Thomson, the value of x / e is around 133 units (C.G.S.) for air at atmospheric pressure. On the other hand, one can infer the value of γ for Boltzmann's experiments. That physicist found that for air:

$$\gamma p = 0.00059,$$

when p is the pressure that corresponds to 760 mm of mercury, i.e, when p is equal to 1033×980 force units (C.G.S.), so:

$$\gamma = \frac{0.00059}{980 \times 1033} = 0.0000000004.$$

Finally, one then has:

$$\frac{\Delta v}{v_0} = \frac{1}{8\pi}\overline{133}^2 \times 0.0000000004 = 0.00000031.$$

The electrical dilatation of gases other than air can be calculated as one does for air, or rather, one can deduce it from what one finds for air upon remarking that the electrical dilatation relate to each other like the values of γ, and consequently, like the values of $\gamma\ p$, where p is the atmospheric value. According to Boltzmann, the values of γp are given for various gases in the following table:

Air………………………...	0.00059
CO_2 ………………………	0.00089
H………………………….	0.00025
CO………………………...	0.00065
Az_2 O (†)………………….	0.00094
Olefiant gas (††)…………..	0.00124
Swamp gas (†††)…………	0.00089

(†) Translator: Now denoted N_20 (i.e., nitrous oxide), since "azote" was the old name for nitrogen.
(††) Translator: Now called ethylene.
(†††) Translator: i.e., methane.

A clever German experimenter, Quincke ([1]), has tried to confirm the phenomenon of the electrical contraction of gases. He worked with air and then with carbonic acid. Those gases were enclosed within a glass container that contained a condenser and was equipped with a small alcohol manometer. Quincke observed a contraction for carbonic acid but saw no motion with air. That negative result for air might point to a flaw in the sensitivity of the manometer ([2]) and the relative weakness of the contraction in the case of air. One sees from the preceding table that for air, the contraction for the electric acid (?) is only 59 / 89 of what it is for carbonic acid.

In 1863, Sir W. Thomson ([3]) observed while charging and discharging a condenser with layers of air a large number of times per second with the aid of a pile of 800 Daniell elements, it produced sound. Sir W. Thomson believed that he could attribute that sound to the contraction that is experienced by air under the influence of the electric charge.

V. – Electrical dilatation of solids. Variation in dielectric strength produced by mechanical stress.

We again apply the principle of the conservation of electricity at the same time as the equivalence principle to the study of the following phenomena. When one subjects an insulating layer to an electrical influence, and indeed, when it is the insulating layer of a condenser, one will confirm that its dimensions will increase during charging and that they will diminish instantaneously at the moment of discharge. The electrical influence has the effect of dilating the insulating layer parallel to the surfaces of the armatures of the condenser. Volta seems to have observed that phenomenon, and then Govi. More recently, Duter rediscovered it, while giving it a precise interpretation and establishing experimentally that the linear dilatation of a glass layer is proportional to the square of the potential that is acquired by the isolated armature of the condenser. Duter appealed to a Leyden jar whose internal armature was composed of water. When he charged that water, it experienced an apparent contraction that was, in reality, due to the dilatation of the envelope. Righi ([4]) confirmed the results what Duter had obtained by a more direct method. Righi took the insulating layer to be a glass tube whose two faces were covered with metallic armatures and which formed a long tubular Leyden jar. When he charged that Leyden jar, it lengthened, and it shrank instantaneously at the moment of discharge. Righi confirmed those variations of length with the aid of a sort of optical comparator.

Let l be the length of Righi's tubular Leyden jar when the potential of the insulating layer is x and the tube is, at the same time, subjected to a tension in the direction of its length that is exerted by a weight p. We take x and p to be the independent variables. Let m be the charge on the isolated armature. Set:

([1]) Wiedemanns Annalen, (1880).

([2]) In a manometer with a finite tube, such as the one that Quincke employed, an imperceptible variation in the curvature of the meniscus will suffice to produce a variation in the pressure of the same order as the one that one wishes to measure here.

([3]) Cosmos, t. XXIII, pp. 519.

([4]) Comptes rendus des séances de l'Académie des Sciences, **87** (1878), pp. 828 and 1036.

(1) $$dm = c\,dc + h\,dp\,,$$

in which *dm* is the quantity of electricity that is received by the internal armature, *c* is the capacity of the condenser, and *h* is a coefficient that can be zero. The principle of the conservation of electricity is expressed by writing that the expression for *dm* is an exact differential. One will then have:

(α) $$\frac{\partial c}{\partial p} = \frac{\partial h}{\partial x}.$$

It remains for us to express the equivalence principle. To that end, we point out that if the glass tube is subjected to an elongation *dl* during which the tensing weight is equal to *p* then the external work produced by a series of elongations and contractions will be equal to $\int -p\,dl$. On the other hand, if the isolated armature is connected to the reservoir whose potential is *x* while the charge on the condenser increases by *dl* then the quantity of electrical energy that is lost by the system of successive reservoirs employed will be equal to $\int -x\,dm$. The equivalence principle demands that those two quantities must be equal for a closed cycle, or that one will have:

$$\int -p\,dl = \int -x\,dm.$$

In other words, when one sets:

$$d\mathcal{E} = p\,dl + x\,dm,$$

one will have that $d\mathcal{E}$ is an exact differential.

In order to express $d\mathcal{E}$, set:

(2) $$dl = a\,dx + b\,dp\,,$$

in which *a* is a coefficient that measures the elongation that Righi observed and *b* is the elasticity coefficient of the tube under the experimental conditions. Those two coefficients *a*, *b* are not mutually independent either. Indeed, we suppose that the tube experiences no permanent deformations. Moreover, whenever *p* and *x* take the same values twice, *l* will likewise assume its original value. Furthermore, *l* is a function of *x* and *p*. As a result, the expression for *dl* will be an exact differential and one will have:

(3) $$\frac{\partial a}{\partial p} = \frac{\partial b}{\partial x}.$$

Upon replacing *dl* with its value, one will have:

$$-\,d\mathcal{E} = (ap + cx)\,dx + (bp + hx)\,dp\,.$$

In order for $d\mathcal{E}$ to be an exact differential, it is necessary that one must have:

$$\frac{\partial(ap+cx)}{\partial p} = \frac{\partial(bp+hx)}{\partial x} .$$

Upon developing that equation and taking equation (3) into account, one will have:

$$(\beta) \qquad a = x\left(\frac{\partial h}{\partial x} - \frac{\partial c}{\partial p}\right) + h .$$

That equation expresses the equivalence principle. Upon combining it with equation (α), it can be simplified. One finally obtains the system of two distinct and compatible equations:

$$(\alpha) \qquad \frac{\partial c}{\partial p} = \frac{\partial h}{\partial x} ,$$

$$(\beta') \qquad a = h .$$

From the experiments of Duter and Righi, a is positive, so h will be non-zero and positive. Now, h is the partial derivative of m with respect to p. The physical interpretation of that result is as follows: Once the tubular jar has been charged to a constant potential x, it will suffice to increase the tension weight p in order to produce a reduction in the quantity of free electricity, in other words, to reduce the electrical capacity of the condenser. Everything happens as if the dielectric strength of the insulated layer will increase when one subjects it to an increasing mechanical tension.

From the experiments of Duter and Righi, the elongation Δl that is due to the potential x is proportional to x^2. One will then have:

$$(4) \qquad \Delta l = \tfrac{1}{2} K x^2 ,$$

in which K is a constant. Let us introduce that experimental result into our analysis. One infers from equation (4) that:

$$a = \frac{\partial l}{\partial x} = K x ,$$

and since one has $h = a$, $h = K x$. As a result, $\partial h / \partial x = K$, and finally, by virtue of equation (α), namely, $\partial h / \partial x = \partial c / \partial p$, it will follow that $c = K p + c_0$, where c_0 is the value of the capacity when the tension weight is zero. Thus, from the fact that the observed elongation is proportional to the square of the potential, one can conclude that the electrical capacity will vary in proportion to the tension weight.

The analysis then indicates the existence of a phenomenon that experiment has yet to exhibit: When one subjects the insulating layer to an increasing mechanical tension, its dielectric strength will increase in proportion to the tension.

Is the electrical dilatation of glass due to the variation of the elastic coefficients of that substance or to the direct action of the electrification? The latter explanation is the true one. Indeed, one has seen from experiments that $a = K\,x$, where K is a constant. Thus $\partial a / \partial p$ will be equal to zero identically. Hence, ultimately, from equation (3), the same thing will be true for $\partial c / \partial x$, i.e., the elastic coefficient b will be independent of the electrification.

VI. – Electrification of hemihedral crystals by compression. Deformation of those crystals produced by electrical influences.

One can apply the same analytical procedure to the phenomenon that was recently discovered by P. and J. Curie ([1]). When one compresses tourmaline along one of its axes, one will observe that its two bases have been electrified in opposite senses. The base A, which will become positive by the effect of heating, will become positive by the effect of pressure. In the course of decompressing the crystal, the free electricity will once more disappear. It will appear and disappear instantaneously at the same as the pressure. From the Curies, the quantity of free electricity that appears will be proportional to the weight p that one lays upon the tourmaline in order to compress it, and independently of the dimensions of the crystal. Other crystals such as quartz and topaz will behave like tourmaline when one compresses them along a hemihedral axis.

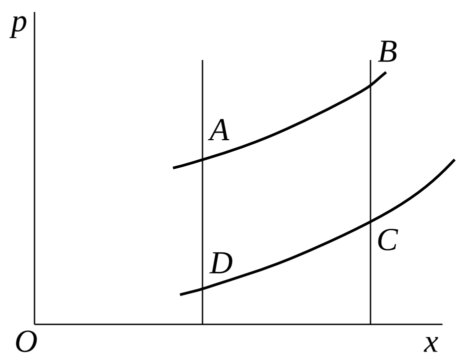

Figure 2.

Suppose that the bases of a tourmaline crystal are equipped with metallic armatures, one of which B is connected to ground, while the other one A can remain isolated or connected with an electric reservoir. One can then vary the pressure p and the potential x of the armature A and make that armature traverse a closed cycle. Take p and x to be independent variables. Let Op, Ox be two rectilinear coordinate axes (Fig. 2). One can represent the state of A at any moment by the position of a representative point P whose coordinates are p and x. If one varies p while keeping A connected to an electrical reservoir of invariable potential then A will experience a variation at constant

([1]) Bulletin de la Société météorologique de France, 1880.

potential that is represented by a line such as *AD*, *BC* that is parallel to the pressure axis. If one varies p while leaving A then electricity will develop or disappear at A, and consequently, the potential will vary with p, and one will have a variation that is represented by a curve such as *AB*, *CD*. One can form a closed cycle *ABCD* that is traversed by the point P in one sense and the other out of two curves of one type and two curves of the other.

One can then apply the principle of conservation of electricity and the equivalence principle to the Curie phenomenon.

Let dm be the quantity of electricity that is received by the armature A when x increases by dx and p increases by dp. Set:

$$dm = c\, dx + h\, dp\,, \tag{1}$$

in which c is what one can call the capacity of A at constant pressure, and h is a coefficient that is negative if, as we suppose, the armature A is applied to those of the bases of the tourmaline that is positively electrified by the pressure. The principle of the conservation of electricity is expressed by the equation:

$$\frac{\partial c}{\partial p} = \frac{\partial h}{\partial x}. \tag{α}$$

In order to express the equivalence principle, let l denote the length of the crystal and set:

$$dl = a\, dx + b\, dp\,, \tag{2}$$

in which b is the elastic coefficient of the crystal and a is a coefficient that we suppose to be non-zero. The rest of the calculations are performed as in the case of the phenomena that were presented for Righi's tubular Leyden jar, except for a change of sign. Here, p represents a pressure, rather than a tension, as it did before. One easily finds that the two principles of conservation and equivalence are expressed by the system of equations:

$$\frac{\partial c}{\partial p} = \frac{\partial h}{\partial x}, \tag{α}$$

$$a = -h\,. \tag{β'}$$

Since h is non-zero and negative, it would follow that a is positive, and consequently, upon referring to equation (2), that l must increase with x. Hence, if one electrifies tourmaline by positively charging its base A then the crystal will *elongate*. The importance of that phenomenon should be pointed out, because the attraction that is produced between the opposite charges that accumulate on the bases tends to produce a contraction: The elongation of the crystal is then a change of structure that is produced by the electrical influence.

According to the Curies, the quantity of electricity that is released by the compression of tourmaline is proportional to the variation in pressure p, and is independent of the dimensions of the crystal, moreover. One will then have:

$$-\frac{\partial m}{\partial p} = K \qquad \text{or} \qquad -h = K ,$$

in which K is a positive constant. One then has the two consequences:

1. From equation (β'), one has:

$$a = -h ,$$

and consequently:

$$a = K ,$$

and since from equation (3), a is nothing but $\partial l / \partial x$, it will ultimately suffice to have:

$$l = K x + l_0 ,$$

in which l_0 is the length of unelectrified tourmaline. The electrification then produces an elongation that is proportional to the potential.

2. Since h is constant, if will follow that the derivative $\partial h / \partial x$ is zero. Hence, from equation (α), the same thing will be true for the derivative $\partial c / \partial p$. Thus, c is independent of p. The capacity of a condenser with a tourmaline layer is then independent of the compression that one subjects the crystal to.

VII. – Pyroelectric phenomena. Cooling produced by electrification.

In all of the electrical or mechanical actions that were considered above, we supposed that the temperature remained invariable. We now propose to study the phenomena that can be produced when we vary the temperature.

In the first place, consider the pyroelectric properties of tourmaline. One knows that when one heats tourmaline, one of its bases (the one that we previously called A, which is also the one that presents the most acute solid angles) will become charged with positive electricity. Suppose that we have equipped the two bases of the tourmaline with metallic armatures, with the armature at A being isolated, while the armature at B is connected to ground. Take the independent variables to be the potential x and the absolute temperature T. Set:

$$dm = c\, dx + h\, dT , \tag{1}$$

in which dm is the quantity of electricity that is received by A when x increases by dx and T increases by dT, c is the electrical capacity of the armature A, and h is a negative coefficient. h is the quantity of electricity that is released by an elevation of temperature that is equal to unity.

The principle of conservation of electricity is expressed by the equation:

$$(\alpha) \qquad \frac{\partial c}{\partial T} = \frac{\partial h}{\partial x}.$$

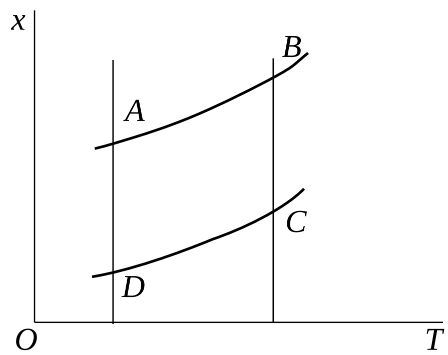

Figure 3.

One can combine that equation with one that expresses the equivalence principle. One must first show that there is good reason to apply that principle here. To that end, it will suffice to show that one can appeal to a tourmaline crystal in order to transform the heat into electrical energy while the crystal traverses a closed cycle. Let T and x be the abscissa and ordinate along two rectangular axes OT, Ox (Fig. 3). When one raises the temperature, while the armature A remains isolated, the potential x will increase, and the variation of that potential will be represented by a line such as AB, DC. If one caries the temperature while keeping A connected with an electrical reservoir then the potential will remain constant, and its variation will be represented by a line that is parallel to the potential axis, such as BC or AD. One can form a quadrilateral that represents a closed cycle with two lines of the first type and two lines of the second type. If the cycle is traversed in the sense $ABCDA$ then the electricity that is taken from the reservoir whose potential is represented by the ordinate AD (that reservoir can be the Earth) will be found, by definition, to be transported into the reservoir whose potential is represented by the ordinate of BC. The electrical energy was thus created at the expense of a certain quantity of heat because the cycle is closed, and heat is the only type of energy that comes into play. One must then have:

$$E\int dQ = \int x\,dm$$

for a closed cycle, where the left-hand side represents the quantity of heat that is absorbed, when expressed in units of work, and the right-hand side represents the electrical energy that is produced. E is the mechanical equivalent of heat. If one sets:

$$d\mathcal{E} = x\,dm - E\,dQ$$

then $d\mathcal{E}$ must be an exact differential. Set:

$$dQ = a\,dx + b\,dT\,,$$

in which b is a quantity that is specific to the layer, and a is a constant that can be zero. Upon substituting the values of dm and dQ, the expression for $d\mathcal{E}$ will become:

$$d\mathcal{E} = (c\,x - E\,a)\,dm + (h\,x - E\,b)\,dT\,.$$

In order for $d\mathcal{E}$ to be an exact differential, it is necessary that one must have:

$$(\beta) \qquad x\left(\frac{\partial c}{\partial T}-\frac{\partial h}{\partial x}\right)-E\left(\frac{\partial a}{\partial T}-\frac{\partial b}{\partial x}\right) = h\,.$$

That equation expresses the principle of the conservation of energy. We can simplify it by applying Carnot's principle.

That principle applies to any closed, reversible cycle in which heat is transformed into mechanical work. Here, heat is transformed into electrical energy, but we point out that once electrical energy is produced, it can be, in turn, transformed integrally into mechanical work, and conversely, in such a way that one if one adds a reversible electric motor to the system that is composed of tourmaline and the reservoirs then one will have constructed a reversible thermal motor. One can then apply Carnot's principle. It will then follow that dQ / T is an exact differential. One will then have the integrability condition:

$$\frac{a}{T} = \frac{\partial a}{\partial T}-\frac{\partial b}{\partial x}\,.$$

Upon replacing the right-hand side with its value in equation (3), one will get:

$$(\beta') \qquad x\left(\frac{\partial c}{\partial T}-\frac{\partial h}{\partial x}\right)-E\,\frac{a}{T} = h\,.$$

Equation (β') expresses the principle of the conservation of energy while taking into account Carnot's principle.

Finally, if one takes equation (a) into account then one will see that the term in x will disappear from equation (β') and that it will reduce to:

$$(\beta'') \qquad \frac{Ea}{T} = -\,h\,.$$

What is the physical interpretation of that result? Since h is a negative quantity, it will result from equation (β'') that a is a positive quantity. Now, a is the partial derivative of the quantity of heat Q with respect to the potential x. If one then positively electrifies the pole A of tourmaline at constant temperature then heat will be absorbed, or rather, the crystal *will be cooled.* If one similarly electrifies the pole B then the opposite effect will be produced.

From Gaugain's experiments, the quantity of electricity that is produced by heating or cooling tourmaline will be simply proportional to the variation in temperature. In other words, the value of $-h$ is a positive constant number k. It will then follow that one has $\partial h / \partial x = 0$, and consequently, from equation (α), $\partial c / \partial T = 0$. The electrical capacity of a condenser with a layer of tourmaline is then independent of the temperature.

Is that true in general? If one substitutes a different body, such as a layer of glass, for the tourmaline plate, will the capacity of the condenser thus-formed be once more independent of temperature? Only experiments can resolve that question. Suppose that one experimentally confirms that the capacity of a condenser with a glass layer varies; for example, it might vary with temperature. That variation would constitute a new phenomenon that one can, in turn, subject to the method of calculation that was previously employed. One will then find that if the insulating layer has an electrical capacity that increases with temperature then it must enjoy the opposite property, namely, that it will *cool* when one subjects it to the influence of an electrified body.

VIII. – General remarks. Reciprocal phenomena. Extension of Lenz's law.

The examples that I just gave suffice to show how one can apply the principle of conservation of electricity, at the same time as the equivalence principle, to the analysis of a given electrical phenomenon. No matter what problem one applies those calculations to, one can make the following remarks:

1. The two principles produce two distinct and compatible condition equations.

That fact should not be surprising, since the two principles are themselves distinct and compatible.

2. The system of two equations thus-obtained is by two physical laws, one of which defines a new physical phenomenon that is the reciprocal of the given phenomenon.

3. The principle of conservation of electricity is necessary if one is to establish those conclusions. The equivalence principle, taken by itself, would not suffice. Notably, it would not suffice to prove the existence of the reciprocal phenomenon.

In regard to that, it might be useful to point out that one must be wary of confusing what one can call the *reversibility* of a phenomenon with the existence of a *reciprocal* phenomenon. They are two distinct ideas that are logically and physically separable. A reversible phenomenon is one that takes place in one sense or the other indifferently. Although it is reversible, it might not have a *reciprocal*. The reciprocal phenomenon is a new action in which there is not simply a change of sign, but a permutation of the cause and effect. Some examples will make that distinction clearer.

The action of a current on a magnetized needle is a reversible phenomenon: The deviation of the needle varies in a continuous manner, so it goes to zero and changes sign with the current. However, whether the deviation is to the right or the left, it is always the same phenomenon. Its reciprocal would be the production of an electromotive force of induction that is due to the motion of the needle, i.e., a different phenomenon.

The deviation of the plane of polarization of light under the action of a constant electrical current is also a reversible phenomenon, even though one does not know of its reciprocal. One has never confirmed that a rotation of the plane of polarization would give rise to an electric current in a neighboring circuit.

The Boltzmann phenomenon, namely, the variation of the dielectric strength of a gas with pressure, is a reversible phenomenon. We have proved the existence of a reciprocal phenomenon, which is the electrical contraction of the gas. Similarly, in each of the problems that were treated above, we have assumed the reversibility of the phenomenon under study and proves the existence of the reciprocal phenomenon by calculation.

The sense of the reciprocal phenomenon is found by a very simple rule: That sense is always such that *the reciprocal phenomenon will tend to opposite to the production of the original phenomenon.*

Indeed, one sees that the compression of a gas will diminish the electrical potential of a conductor that is placed in that gas. Reciprocally, the reduction of the potential will tend to produce a dilatation of the gas, and consequently, to prevent the motion of a compressor piston. Similarly, upon putting a weight on tourmaline, one will produce electrification of the crystal. Reciprocally, that electrification has a sense such that it gives rise to mechanical forces that oppose the contraction of the crystal. When one provides electricity to a glass layer, it will dilate. By virtue of the reciprocal phenomenon, a reduction in the electrical capacity will be produced that tends to prevent the electrification. Finally, when one heats tourmaline, the crystal electrifies. Reciprocally, that electrification has a sense that produces absorption of heat and consequently tends to prevent heating. Lenz's law, which governs the phenomena of induction, is obviously included in the rule that was just given.

When one assumes the existence of a reciprocal phenomenon, the rule that permits one to predict the sense is a consequence of the equivalence principle. Indeed, the reciprocal phenomenon is the mechanism by which one form of energy is absorbed and transformed into electrical or mechanical energy. However, in order to establish *the existence* of the reciprocal phenomenon, as when one does when finding the law, one has seen that the equivalence principle is not sufficient and that one must combine it with the principle of the conservation of electricity.

Note A.

In order for an integral such as $\int dm$ to be zero whenever it is extended along a closed cycle, it is necessary and sufficient that the quantity that is placed under the $\int$ sign should be an exact differential. That theorem was applied to thermodynamics by Sir W. Thomson and G. Kirchhoff.

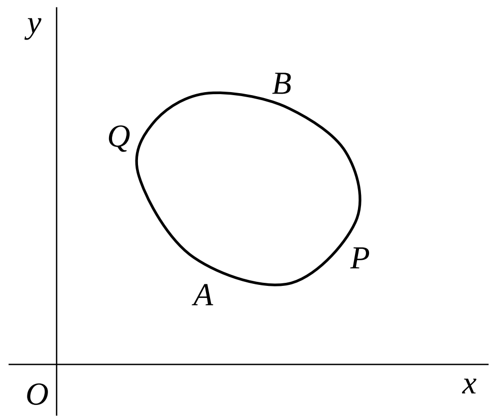

Figure 4.

Indeed, let x and y be independent variables. Let Ox, Oy be two axes of a rectilinear coordinates (Fig. 4). Take x and y to be the coordinates of a moving point P with respect to those axes. When x and y vary continuously, the point P will describe a continuous curve that will be closed when the cycle is closed. By hypothesis, the value of the integral is zero in that case. Consequently, the value of the integral that is obtained by going from one point of the curve A to any other point B along the arc APB is equal and opposite in sign to the value that is obtained by going from the point B to the point A along the arc BQA. Consequently, when one goes from the point A to the point B by either the arc APB or the arc AQB, one will obtain integrals whose values are equal and have the same sign. Now, those arcs are arbitrary. The value of the integral will then be independent of the path traversed. It will depend upon only the positions of the extreme points A and B. Thus, if one represents the coordinates of an arbitrary point B in the plane by x and y then the integral will be a function of x and y when it is extended up to that point. The quantity that is placed under the $\int$ sign is then the change in that function. In other words, it is an exact differential; that is what had to be proved.

Conversely, if dm is the differential of a function of x and y then the definite integral $\int dm$ will represent a quantity of electricity, and when that quantity is extended over a closed cycle, it will always be zero; that is what experiments teach us. From the theorem that was just recalled, one can express that same experimental fact by writing that the integrability condition is satisfied.

Note B.

When a certain quantity of mechanical work $\mathcal{T}$ is produced in a phenomenon or a series of phenomena, and at the same time, the electrical energy that is contained in a system of electrical reservoirs is subjected to a reduction that is equal to $\int x\,dm$, I will say that one has:

$$\mathcal{T} = \int x\,dm,$$

no matter what the nature of the phenomenon might be, and with the single condition that the cycle traversed should be closed, i.e., the condition that, by definition, no other phenomena are produced than the ones that are measured by the terms $\mathcal{T}$ and $\int x\,dm$, and that the final state of the system is identical at any point to its initial state.

Indeed, I can reduce the electrical reservoirs to the initial state by appealing to an electrical influence machine, and on the condition that one must expend a quantity of work $-\mathcal{T}'$ in order to make that machine function. Having done that, the total production of work will be reduced to $\mathcal{T} + \mathcal{T}'$. I say that this quantity is necessarily zero. Indeed, the entire system, which is comprised of the set of all electrical reservoirs this time, is reduced to its initial state precisely, so the same series of operations can be repeated indefinitely. Consequently, since the quantity $\mathcal{T} + \mathcal{T}'$ is positive, one would have realized perpetual motion. If it were negative then one would have a mechanism that destroy an indefinite quantity of work without producing its equivalent.

One must therefore necessarily have:

$$\mathcal{T} + \mathcal{T}' = 0 \qquad \text{or} \qquad \mathcal{T} = -\mathcal{T}'.$$

On the other hand, one knows that the quantity of work $-\mathcal{T}'$ that is necessary to make the influence machine function is equal to $\int x\,dm$ identically; that is an identity that results from the definition of the potential x. As a result, one has $\mathcal{T} = \int x\,dm$; that is what was to be proved.

Similarly, if a certain quantity of mechanical work is absorbed during a series of arbitrary phenomena then the quantity of work that is absorbed will be equal to the increase in the electrical energy, with the single condition that the cycle should be closed. One proves that by the same argument by supposing that the influence machine acts as a driving machine.

That proof does not suppose that the phenomenon considered is reversible.

The theorem that was just proved can be translated into an integrability condition. Indeed, upon replacing $\mathcal{T}$ with the symbol $\int d\mathcal{T}$, one has $\int d\mathcal{T} = \int x\,dm$ or $\int (d\mathcal{T} - x\,dm) = 0$, for a closed cycle. That amounts to writing (as one saw in the previous Note) that the quantity that is placed under the $\int$ is an exact differential.

Note C.

The two principles of the conservation of electricity and the conservation and the conservation of energy apply to the analysis of electrocapillary phenomena that are presented by an electrode of mercury. As I have shown elsewhere ([1]), one can deduce two relations, the first of which is found to be confirmed by recent experiments that are due to R. Blondlot.

([1]) *See* Annales de Chimie et de Physique (1875).

That first relation can be written:

$$c = -\frac{\partial^2 A}{\partial x^2}, \tag{1}$$

in which c is the polarization capacity per unit area for a mercury electrode. A is the capillary constant of mercury, and x is the potential difference between that metal and the liquid that it is immersed in. A is a function of x, and in 1877 I showed ([1]), by means of experiments that Blondlot and myself then extended to a great number of liquids, that the function A is the same no matter what the nature of the liquids employed. The same thing must then be true for its second derivative, and consequently for the capacity c.

Blondlot ([2]) showed experimentally that the polarization capacity of *platinum* is independent of the nature of the liquid that it is immersed in, provided that one keeps the value of x constant. As noted by Blondlot, that law can be predicted from formula (1) without nonetheless being a necessary consequence. Indeed, he did not prove that platinum (which is a solid body) has a capillary constant, and consequently that formula (1) applicable. The law that Blondlot discovered can then be proved only experimentally. However, once it has been proved, that law will be an indirect confirmation of formula (1), and consequently of the principle of the conservation of electricity.

([1]) *Ibid.*, 1877.

([2]) Doctoral thesis, J. de Phys. (1881).