"Sopra l'elettrone rotante," Nouv. Cim. (8) **3** (1926), 226-235.

On the rotating electron

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The measurable elements of the electron are its electric charge and its mass. As is known, in classical electrodynamics, one attempts to give an interpretation for the latter that reduces it to the former. Thus, one considers the electron to be a distribution of electricity over a surface or volume with spherical symmetry whose electromagnetic mass is generally identified with the total mass of the electron. Despite these ideas about its structure, in the theory of the atom, the electron has almost always been considered to be a material point up to now. It was only in recent years that Uhlenbeck and Goudsmit ([1]) made the hypothesis that the reason for some spectroscopic phenomena – in particular, the anomalous Zeeman effect – was to be found in a structural element of the electron. Those authors assumed precisely that the electron is animated with a rotational motion around itself, in such a way that it possesses a quantity of areal motion, namely, a magnetic moment. The present work is dedicated to a discussion of that hypothesis, and in particular, it is shown that it is very probable that the electron can be assigned linear dimensions that are notably larger than what have been considered up to now, which has been confirmed by the ultimate experimental facts.

Qualitatively, the Zeeman phenomenon finds its interpretation in what one calls *Larmor's theorem*, which says: The perturbation that is produced in the motion of a mechanical system that is comprised of material points that all have the same mass m and the same electric charge e and are in a uniform magnetic field of intensity H consists, in the first approximation, of a uniform precession of the entire system around the direction of the field with a frequency of $\nu_L = eH / 4\pi mc$. Larmor's theorem is intimately linked with the result that for a system of the type considered, there is a relationship between the magnetic moment and the mechanical one that depends upon only the charge and mass of the points and is given by $e / 2mc$, precisely.

If the electrons of an atom are considered to be material points then Larmor's theorem can also be applied to the atom since its nucleus can be considered to be closed. Therefore, from that viewpoint, it can happen that the frequencies of the rays that are emitted from the atom in the magnetic field are combinations of the proper frequencies of the unperturbed atom and the Larmor frequency. Any ray of the unperturbed atom with a frequency of ν can therefore correspond to three rays with frequencies $\nu - \nu_L$, ν, $\nu + \nu_L$. It is known that this normal form for the Zeeman effect is observed for only a very limited number of rays. Rather, in the majority of cases, one has a decomposition into more than three components whose separation is generally different from the normal one;

([1]) Uhlenbeck and Goudsmit, Naturwiss. **13** (1925), 953; Nature **20** (Feb, 1926); Bohr, Nature, *ibid.*

that is what is called the *anomalous Zeeman effect.* In addition, one finds that while the displacement of any component of the position of the unperturbed ray is proportional to H for a weak magnetic field, as one increases the field, a deformation of the Zeeman configuration will come about, in such a way that it will tend to be transformed into the normal triplet in the limit of very strong fields (viz., the *Paschen-Back effect*).

From the correspondence principle, as well as in Bohr's theory, the frequency of the ray that is emitted by the perturbed atom in a magnetic field can be calculated as the combined frequency of the proper frequencies of the atom and the frequency of precession; it is therefore clear that in order to account for the anomalous Zeeman effect, one must assume that the velocity of precession of the atom is different from the normal one, which must be considered to be the limit in a very strong field.

An atomic model that seeks to account for this peculiarity, if only incompletely and unsatisfactorily in several regards, is the one that was developed by Landé. Landé ([1]) distinguished the luminous electrons in the atom, which are the ones that emit rays and generally move in an orbit that is quite far from the nucleus, from the totality of all other electrons that are closer to the nucleus, i.e., the *core* (Ger: *Rumpf*). Each of these two elements possesses a quantity of areal motion that Landé called K and R, respectively. The resultant J of K and R is the quantity of areal motion of the entire atom, which will keep an invariable direction in the absence of external forces. Forces act between an electron and the core that depend upon the orientation of the latter with respect to the plane of the electronic orbit – i.e., the angle between K and R. Those forces are such that they give rise to a precession of the entire atom around the axis J. That precession has a characteristic effect on the structure of the emission spectrum of the unperturbed atom, and in fact, it is clear that the frequency of any ray that the atom emits that does not have the given precession will combine with the frequency of the precession in such a way that it gives rise to a multiple ray.

We shall study what the effect will be of placing that atom in a magnetic field. If one assumes that, like the luminous electron, the core is subject to precessing around the field with the normal Larmor frequency then the effect will consist of imprinting that precession on the entire atom, and the final result will be found to be the normal Zeeman effect. In order to explain the anomalous Zeeman effect and the Paschen-Back effect, Landé assumed that the luminous electrons are subject to precession with the normal frequency, while the core is subject to twice the frequency. In order for that to be true, it is also necessary to assume that the ratio of the magnetic moment and the mechanical moment of the luminous electron is that of the normal one, while it is double for the core. Until the force that is exerted between the electrons and the core is large in comparison to the force that is exerted by the field H, the motions of the core and the electrons will remain coupled, in such a way that all of the atom will precess with a frequency that is between the two. For weak fields, one will therefore have a precession, and therefore an anomalous Zeeman effect. On the contrary, when the action of the field dominates the interaction between the electron and the core, each of those elements will precess independently of the other with its own frequency of precession, and since that of the electron is normal, the normal Zeeman separation will result for strong fields – i.e., the Paschen-Back effect. That schema accounts for the characteristic principles of the

([1]) Landé, Zeit. Phys. **15** (1923), 189.

observed anomalous Zeeman effect ([1]), not just qualitatively, but also quantitatively. Despite those successes, it is always regarded as insufficient and provisional, because since the core is comprised of only electrons, it does not explain why its frequency of precession must be twice that of the normal. Another inconvenience of Landé's theory is the following one: If R is interpreted as the moment of the core then one will need to assume that its value coincides with the total moment J' of the atomic ion, since the ion is precisely what remains of the atom when one removes the luminous electrons. However, one finds that any well-defined value J' for the moment of the ion can correspond to two values of R that differ from J' by $\pm 1 / 2$.

It was precisely in the hopes of avoiding those inconveniences that Uhlenbeck and Goudsmit introduced the hypothesis of the rotating electron. Observe that in Landé's theory the interpretation of R as the moment of the core is somewhat arbitrary. Despite all of its inconveniences, it was probably chosen due to a lack of other elements in the atom to which one could attribute R when one started with the hypothesis that one should neglect the structure of the electron *a priori*, and that was, however, where Uhlenbeck and Goudsmit looked for the meaning of R. They assumed precisely that the electron rotated around itself and therefore possessed a mechanical moment, as well as a magnetic moment. Naturally, for a system of that type, the ratio of the magnetic moment to the mechanical one will depend upon the distribution of the charge and mass; in order to be in agreement with experimental facts, one assumes that this distribution is such that it gives a ratio that is twice that of Larmor. One also assumes that the state of rotation of all the electrons is the same, and that it differs only by the different orientations that must be determined from the quantum relationships with the rest of the atom and any possible external field. R is interpreted as the vector sum of all the moments of the electrons in the atom.

One has the following advantages of this interpretation:

1) The fundamental difficulty in the magnetic anomaly of R disappears, since in fact, for the rotational motion of any electron, the ratio of the magnetic moment to the mechanical moment is twice the Larmor ratio, so the electron in an external magnetic field will be subject to a precession with a frequency that is twice the normal one.

2) One also understands how one can get diverse values for R when one adds a new electron to a positive ion in such a way as to form a neutral atom, according to the orientation of that electron with respect to the ion; in fact, that is found to be the case.

3) It accounts for the situation that was observed by Stoner ([2]) and Pauli ([3]) that in order to be able to construct a unified model for the successive formation of the elements in regard to their spectroscopic properties, one must attribute a different degree of

([1]) The Landé schema is valid only for things that pertain to spectra of the first degree (Ger. *erste Stufe*); there are generalizations for the other ones [Russell and Sounders, Astr. Journ. **61** (1925), 38; Heisenberg, Zeit. Phys. **32** (1925), 841]. However, for our purposes, it is enough to limit ourselves to the case of first-degree spectra.

([2]) Stoner, Phil. Mag. **48** (1924), 719.

([3]) Pauli, Zeit. Phys. **31** (1925), 765.

freedom to the electron from that of point-like matter, and whose origin has been incomprehensible, up to now.

4) In the preceding theorem, the spectrum of hydrogen occupies a singular position in the periodic table, and is interpreted in a manner that is completely different from the spectra of the other atoms with only one valence electron (viz., the alkali metals). However, with the new schema, one establishes a complete analogy, inasmuch as even though the conclusions of Sommerfeld's relativistic theory remain unaltered, one must modify his nomenclature ([1]). That implies a modification of the selection rules and the calculation of the intensities of the various fine structure components, which have shown to be in best agreement with the results of measurement.

5) The parameters of the new theory account for the structure of the Roentgen levels, and in particular, for the fact of the general validity of the relativistic formula for the calculation of the spectral separation, which was inexplicable up to now.

From what we have said, one sees that the hypothesis of the rotating electron illuminates several of the points that were previously quite obscure in the theory of the atom. However, there are various objections that one can raise against it: The first of them was presented by its authors themselves. In effect, one starts with a particular model of the electron that supposes a spherical surface distribution of electricity, and whose mass is calculated as if it were due to only its electrostatic energy. The rotational motion of that system is calculated from the ordinary rules of the quantization of a rotator. In that way, one effectively finds a ratio between the mechanical moment and the magnetic moment that is twice Larmor's normal ratio. However, one finds that the peripheral velocity at the equator will prove to be noticeably larger than the velocity of light. Another inconvenience of the hypothesis of the rotating electron was pointed out by Kronig ([2]). It is known that the nuclei of atoms generally contain electrons. Except in the particular case in which the magnetic moments of the individual electrons are neutralized, the nucleus must therefore possess a resultant magnetic moment, which must manifest itself externally in a paramagnetism of the atom that depends upon that of the nucleus, although that effect has not been observed.

However, neither of those difficulties, the first of which depends upon a choice of a more specialized model for the electron and the other of which is not insurmountable, because there are no substantial difficulties associated with assuming that the magnetic moments of the nuclear electrons are neutralized, seem as serious as the one that one derives from the following considerations, and it is independent of any particular idea regarding the structure of the electron, to a large extent.

It is essential for spectroscopic applications that the rotating electron should possess a magnetic moment whose order of magnitude is that of the Bohr magneton. There will then exist a magnetic field around the electron that will practically coincide with the one that is due to an ordinary point-like dipole at a distance that is large in comparison to the structure of the electron, while it can have noticeable deviations from that dipole field when the distance has the same order of magnitude as the linear dimensions of the

([1]) Uhlenbeck and Goudsmit, Physica (1925), 266. Sommerfeld and Unsöld, Zeit. Phys. **36** (1926).
([2]) Kronig, Nature 17 April 1926, page 550.

structure above. Therefore, one obtains a lower limit on the magnetic energy of the electron that is calculated from the magnetic energy that is localized in the exterior of a sphere of radius R whose order of magnitude is that of the linear dimensions of the electron and identifying that field with the field of a point-like electron.

The intensity H of the magnetic field of a point-like dipole is given by:

$$H^2 = \frac{\mu^2}{r^6}(3\cos^2\theta + 1),$$

where r represents the distance from the dipole, θ is the colatitude, and μ is the magnetic moment. The magnetic energy that is contained in the volume element $d\tau$ is therefore:

$$\frac{H^2}{8\pi}d\tau = \frac{\mu^2}{8\pi r^6}(3\cos^2\theta + 1)\,d\tau. \tag{1}$$

The energy W that is contained outside of a sphere of radius R is then:

$$W = \int_\tau \frac{H^2}{8\pi}d\tau = \frac{\mu^2}{8\pi}\int_0^\infty\int_R^\pi \frac{1}{r^6}\,(3\cos^2\theta + 1)\,2\pi r^2 \sin\theta\, d\theta\, dr\,,$$

and after integrating, one will find that:

$$W = \frac{\mu^2}{3R^3}.$$

That energy must correspond to a mass that one calculates with the theory of relativity by dividing W by c^2 (one finds a value with that same order of magnitude in classical electrodynamics). The total energy of the electron will certainly be greater, since one must add the magnetic energy that is contained inside of the sphere of radius R to the preceding expression, as well as the energy of the electronic structure.

One then finds a lower limit for the mass of the electron by taking μ to be the Bohr magneton – viz., 0.92×10^{-20}:

$$m = \frac{\mu^2}{3c^2R^3}.$$

From this, taking into account that the electron has a mass of 0.9×10^{-27}, one deduces that a lower limit for the radius R is:

$$R = 3.3 \times 10^{-12}. \tag{2}$$

This value is about 20 times larger than the one that is ordinarily obtained for the electronic radius. In reality, one cannot measure the latter directly. Nonetheless, the inconvenience is serious, because it is known that the nucleus contains a considerable number of electrons. On the other hand, the linear dimensions of the structure of the nucleus are known with sufficient precision from measuring the deviations in alpha particles when they pass through matter, and as is known, they prove to be of order 10^{-12}

cm. As one sees, the two facts seem quite irreconcilable if one does not assume that the electrons that take part in nuclear structure diverge considerably from nature.

It seems that the preceding deduction for the lower limit (2) on the radius of the electron lends itself to difficult and grave objections, because it will break down when one needs to assume the illegitimacy of the calculation of the magnetic energy by formula (1), which seems plausible inside of the electronic structure, where the very notion of magnetic field might lose all significance, but will be difficult to assume outside of that structure. Another way out can consist of not preserving the validity of the relativistic relation between mass and energy, or even assuming that the magnetic structure can be appreciably larger than the electric one.

From this discussion, it seems that, despite the grave energetic difficulties that have been pointed out, one can conclude that the hypothesis of the rotating electron must not be abandoned as a result of them. Naturally, we do not think that it should be taken too literally, in the sense that one should truly imagine the electron to be a macroscopic body that is charged with electricity and rotate around itself, since all that is essential for the applications is that the electron should possess a mechanical moment and a magnetic moment that are independent of the particular model that represents their origins.

At any rate, the question cannot be considered to be resolved as long as there is no further direct experimental evidence that would confirm or contradict the hypothesis of a rotating electron.