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On the nonlinear continuum theory of point defects

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With two figures

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Abstract. – Point defects are described directly by means of quasi-plastic distortion in the context of nonlinear continuum theory. Considering that fact is sufficient for one to derive the equations of motion of both the medium and the foreign matter when one starts from the LAGRANGIAN of conventional elasticity theory.

Introduction

The treatment of lattice defects in the continuum approximation has proved to be sufficient for many problems, as well as being preferable on mathematical grounds.

The perturbation of the phenomena that happen in the vicinity of a lattice defect can be described most simply by the introduction of an intermediate state in a continuum [**1**, **2**]. It will be established by comparing it with an ideal state, namely, the so-called "initial state." In that sense, the lattice defects will be characterized by the initial state-intermediate state transition – i.e., by the corresponding distortion. The exhibition of the connection between those distortions on the one hand and the density of the lattice defects and their properties, on the other, is the actual problem that must be solved in any description of lattice defects in a continuum. That problem is explained in the case of dislocations: The rotation of the plastic distortion is set equal to the dislocation density. In contrast to that, one cannot satisfy the description of continuous distributions of point defects completely [**2**, **3**].

Now, it will be shown in the first section of the present paper that following through on the ideas that were pointed out above will effortlessly lead to the relationship between the density of point defects and quasi-plastic distortion. Hence, it will be possible in the second section to conclude the equations of motion for the host matter and the extra matter from the LAGRANGIAN function of ordinary elasticity theory. The third section contains a glimpse of the differential-geometric description of the quantities that point defects introduce. They will be reduced to distributions of point defects with the help of the connection that was presented in Section 1.

1. – Description of point defects

One can think of the transformation that takes a unit crystal from an initial state that is free of stresses or defects to a final state that is endowed with lattice defects and internal stresses as being decomposed into a plastic (quasi-plastic, resp.) deformation with no reaction forces and an elastic one in a known way by interposing an intermediate state that is still stress-free, but no longer defect-free [**2**]. One describes the deformations by means of distortion tensors that associate the connecting vector between two mass-points in one state with the connecting vector between the same mass-points in another state:

$$dx^k = A^k_\kappa \, dx^\kappa = A^k_K \, dx^K , \qquad dx^\kappa = A^\kappa_K \, dx^K = A^\kappa_k \, dx^k . \tag{1.1}$$

In this, K, κ; k refer to the initial, intermediate, and final states, resp. In general, only the total distortion $A^k_{\ K}$ can be represented as a gradient $\partial x^k / \partial x^K$.

In what follows, we would like to attribute the quasi-plastic distortions to the physical properties of point defects, and in that way, define the geometric structure of the intermediate states that correspond to those point defects. In order to do that, we first consider an isolated point defect. On the basis of its deviating interaction potential, when it is embedded into the ideal lattice, it will cause a displacement of the atoms that are immediately closest to it. The following figures shall illustrate that state of affairs in the example of a substitution atom. The figure on the left shows the ideal lattice. If one thinks of the middle atom as being replaced with an atom of a different sort then the distance to the immediately-close atoms will change, perhaps as is represented on the right in the figure. However, corresponding to the definition of the (stress-free) intermediate state, all of the other distances to the neighboring atoms must remain preserved. That means that the volume elements in the intermediate state will no longer fit together with no gaps or overlaps.

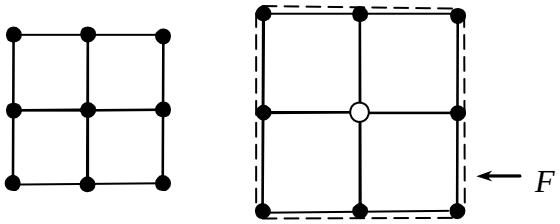

Figure 1. Change in the atomic distances by the insertion of a point defect.

In order to adapt this argument to the continuum picture, one first lays a closed surface F through the atoms that surround the point defect. Now, one can arrange the generation of the intermediate state from the initial state in such a way that the surface will be displaced through the (position-dependent) segment S^κ during the transition from the left-hand side of the figure to the right. Secondly, one must specify the changes in distance inside of the surface. They will be arbitrary, except for the fact that the (physically-given) displacement must carry along the surface S^κ.

In order to do that, one imagines that all of the changes in distance that are required by that displacement take place only in the immediate neighborhood of the surface. The finite displacement S^κ will then take place over an infinitesimal segment (on the surface).

That means: The relative position of two arbitrary neighboring points will remain unchanged, with the exception of the ones that are separate from the surface F, which will be displaced with respect to each other by precisely S^κ. One then has:

$$\begin{aligned} dx^\kappa &= \delta^\kappa_K dx^K + \hat{S}^\kappa \qquad \text{when } dx^K \text{ is rotated through } F, \\ dx^\kappa &= \delta^\kappa_K dx^K \qquad\quad \text{otherwise.} \end{aligned} \tag{1.2}$$

Corresponding to this construction, infinitesimal distances will go over to finite ones in the first case. Accordingly, those conditions can be fulfilled identically when one considers (1.1) by way of the following Ansatz for the quasi-plastic distortion A^κ_K:

$$A^\kappa_K(x^L) \equiv \delta^\kappa_K + \beta^\kappa_K(x^L) = \delta^\kappa_K + \oiint_F df'_K \, \hat{S}^\kappa \delta_A(x'^L - x^L) \tag{1.3}$$

$$[\,\delta_A(x'^L - x^L) = \text{three-dimensional } \delta\text{-function in the initial state}]$$

In order to see that, one substitutes (1.3) into (1.2). If one now considers that one is dealing with a *point* defect (i.e., that its measurements are small in comparison to the separation distances being considered) then one can write, approximately:

$$\beta^\kappa_K(x^L) = \delta_A(x^L - x^L_0) \oiint_F df'_K \, \hat{S}^\kappa \equiv \delta_A(x^L - x^L_0) Q^\kappa_K, \tag{1.4}$$

in the sense of a formal development of the δ-function, in which Q^κ_K is the displacement dipole that KRÖNER introduced [**4**].

Since $\delta_A(x^L - x^L_0)$ is equal to the density of point defects in the initial state $\hat{n}_A$ in the case considered of a singular distribution, one will generally set:

$$\beta^\kappa_K = \hat{n}_A Q^\kappa_K. \tag{1.5}$$

The two special case of para-elastic (dia-elastic, resp.) point defects [**2**] are then characterized by the conditions:

$$Q^\kappa_K = \text{const.}$$

and

$$Q^\kappa_K = Q^\kappa_K(\sigma) \quad \text{with} \quad Q^\kappa_K(0) = 0,$$

respectively.

The desired connection between point defects and the geometric structure of the medium is exhibited by (1.5): The point defects determine the quasi-plastic distortion:

$$A^\kappa_K = \delta^\kappa_K + \hat{n}_A Q^\kappa_K \tag{1.7}$$

by their density of point defects $\hat{n}_A Q^\kappa_K$ (i.e., the usual density of point defects, multiplied by the strength of the individual point defects). In contrast to that, only $A^\kappa_{[K,L]}$ is

established for dislocations. That is based upon the fact that in the first case, the surface of intersection F means the boundary surface of the point defect, while in the latter case, only the boundary line of an intersection surface that is not closed will have any physical meaning.

2. – Equations of motion for a medium with point defects

The starting point for the derivation of the equations of motion is defined by a variational principle. The LAGRANGIAN function of an elasto-plastic medium [**5**, **6**] coincides with that of a purely-plastic one. Corresponding to the split in the total deformation, one must observe that the potential energy depends upon only the elastic deformation $\varepsilon_{\kappa\lambda}^{\text{el.}}$ that refers to the intermediate state. From the argument that was presented in the foregoing section, that is determined by the point defects. The LAGRANGIAN function L will then prove to be:

$$L = T - U = \int d\tau \left\{ \tfrac{1}{2} \rho v^i v_i + \tfrac{1}{2} \hat{\rho} \hat{v}^i \hat{v}_i - \rho U \left(\varepsilon_{\kappa\lambda}^{\text{el.}} \right) \right\}, \tag{2.1}$$

in which $\frac{1}{2} \hat{\rho} \hat{v}^i \hat{v}_i$ can be regarded as the additional contribution of the point defect to the kinetic energy density. In that expression, $\hat{\rho} = \hat{m}\hat{n}$ represents the mass density of the extra matter (i.e., the point defect), ρ is the mass density of the host matter (i.e., the total density minus $\hat{\rho}$), and $\hat{m}$ is the mass of a point defect.

The variation of L comes down to the variation of the positions of the masses and point defect δx^l and $\delta \hat{x}^l$, resp. In that process, we restrict ourselves to para-elastic point defects of only a well-defined type, and thus to Q_K^κ = constant.

That implies (cf., Appendix 1):

$$\left\{ \rho \frac{d}{dt} v_i - \sigma_{i,k}^k + \rho \left(\sigma_m^l Q_l^m \right)_{,i} \right\} \delta x^i + \left\{ \hat{\rho} \frac{\hat{d}}{dt} \hat{v}_i - \hat{\rho} \left(\sigma_m^l Q_l^m \right)_{,i} \right\} \delta \hat{x}^i = 0, \tag{2.2}$$

with Q_l^m as in (A.1.10). In this, $\hat{d}/dt$ (d/dt, resp.) are the substantial time derivatives relative to the point defects (host medium, resp.).

The term $\sigma_m^l Q_l^m$ represents the force on a para-elastic point defect in an elastic stress field that is known from the linear theory [**4**]. In contrast to the theory of dislocations, the equation of motion for point defects possesses an inertial term. If one then eliminates the supplementary term in the first equation of motion then the inertial term will represent the change in the total impulse of the matter (medium plus point defect) that is contained in a volume element due to a force that acts upon its outer surface. Here, we shall avoid a more precise discussion and establish only that σ_i^k can be identified with the total stress field to a high degree of approximation due to the weak concentration of the point defects.

Since the equations of motion are derived from a variational principle, they will be purely reversible. When one considers thermal interactions, the equation of motion for the point defect will be replaced with a diffusion equation of FOKKER-PLANCK type, in which the force on the point defect that appears in its drift term will, in turn, be given by the corresponding term in (2.2).

3. – Differential-geometric description of point defects

Whereas it was shown above how point defects can be described very simply by quasi-plastic distortions, all of the previous attempts at a nonlinear description (cf., e.g., [**2**]) started from differential-geometric methods.

In those methods, the fact was employed that an arbitrary affinity for an arbitrary metric c_{kl} will admit the following decomposition [**7**]:

$$\Gamma^{k}_{nm} = \tfrac{1}{2} c^{ks}\, \partial_{\{n}\, c_{sm\}} + \tfrac{1}{2} c^{ks}\, Q_{\{nsm\}} - c^{ks}\, S_{\{nsm\}}\,, \tag{3.1}$$

with

$$A_{\{klm\}} = A_{klm} + A_{mkl} - A_{lmk}\,, \qquad S_{lnm} \equiv c_{lk}\Gamma^{k}_{[nm]}\,, \qquad Q_{lnm} = -\nabla_l\, c_{nm}\,.$$

Various metric can be used to describe the geometric structure. We first consider the "lattice affinity" (cf., A.2):

$$\hat{\Gamma}^{k}_{mn} \equiv A^{k}_{K}\, A^{K}_{n,m}\,. \tag{3.2}$$

It is symmetric in the case of point defects, due to the fact that $A^{k}_{K} = \partial x^k / \partial x^K$. S^{k}_{nm} will then vanish identically.

If:

$$b_{kl} \equiv A^{K}_{k} A^{L}_{l} \delta_{KL}\,, \tag{3.3}$$

$$g_{kl} \equiv A^{\kappa}_{k} A^{\lambda}_{l} \delta_{\kappa\lambda}\,, \tag{3.4}$$

and δ_{kl} denote the metrics in the initial, intermediate, and final states, resp., then the elastic deformation:

$$\varepsilon^{\text{el.}}_{kl} \equiv \tfrac{1}{2}(\delta_{kl} - g_{kl}) \tag{3.5}$$

and quasi-plastic deformation:

$$\hat{\varepsilon}_{kl} \equiv \tfrac{1}{2}(g_{kl} - b_{kl}) \tag{3.6}$$

will be defined with their help.

If one now chooses the metric of the intermediate state in the decomposition (3.1) then the first sum will be established by the elastic deformations. The second sum must be determined essentially by the point defects, since it will vanish in their absence, due to the fact that:

$$Q_{lnm} \equiv -\hat{\nabla}_k g_{mn} = -2\hat{\nabla}_k \hat{\varepsilon}_{mn}\,. \tag{3.7}$$

In this equation, the fact was used that one must have:

$$\hat{\nabla}_k b_{mn} = 0, \tag{3.8}$$

by definition.

The lattice affinity $\hat{\Gamma}^k_{mn}$ (which is non-metric for g_{kl}) determines an everywhere-single-valued parallel translation. By contrast, the state connection that is defined by:

$$\bar{\Gamma}^k_{mn} \equiv \tfrac{1}{2} g^{ks}\, g_{\{nsm\}} = \hat{\Gamma}^k_{mn} - \tfrac{1}{2} g^{ks}\, Q_{\{nsm\}} \tag{3.9}$$

(which is metric with respect to g_{kl}) does not exhibit any teleparallelism. As a result, the RIEMANNian curvature tensor cannot vanish in all of space:

$$R^k_{nml}(\bar{\Gamma}) = R_{nml}{}^k(\hat{\Gamma} - Q_{\{\}}) \equiv Q_{nml}{}^k, \tag{3.10}$$

with

$$R_{nml}{}^k(\Gamma) = \Gamma^k_{ml,n} + \Gamma^k_{ns}\Gamma^s_{ml} - \Gamma^k_{nl,m} - \Gamma^k_{ms}\Gamma^s_{nl}.$$

The term "matter tensor" has been accepted for $T_{nml}{}^k$, since it must, in turn, include the influence of the point defects.

In the sense of the differential-geometric method, one can consider Q_{knm}, $\hat{\varepsilon}_{mn}$, or $T_{mnl}{}^k$ as being given and determined by just the point defects. However, that is not strictly true, since it is only in the initial state that the quantities:

$$\begin{aligned} \hat{\varepsilon}_{MN} &= \tfrac{1}{2}(g_{MN} - \delta_{MN}), \\ Q_{KMN} &= -\tfrac{1}{2}\hat{\varepsilon}_{MN,K}, \\ T_{NML}{}^L &= R\,(\tfrac{1}{2} g^{KS}\, \partial_{\{N}\, g_{SM\}}), \end{aligned} \tag{3.11}$$

which are defined by:

$$g_{MN} = A^\kappa_K A^\lambda_L \delta_{\lambda\kappa}, \tag{3.12}$$

are given in terms of only the quasi-plastic distortions A^κ_K that characterize the point defect, and for (1.7), they can generally be expressed in a rather complicated way in terms of the density of point defects and dipole strength. By contrast, Q_{km}, ε_{km}, and $T_{mnl}{}^k$ will first follow from (3.11) by converting them with the total distortion $A^k_K = A^k_\kappa A^\kappa_K$, which contains the elastic distortion, as well as the quasi-plastic one.

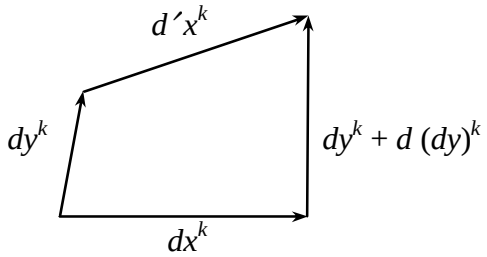

Figure 2. The definition of ZORAWSKI's parallel displacement.

ZORAWSKI [**3**] sought to exhibit a direct connection between the matter tensor and point defects when he attempted to attach an immediate intuitive meaning to the parallel translation that is defined by the state connection (Fig. 2). The displacement of a vector dy^k, which is regarded as infinitesimal, shall result by displacing the end (starting, resp.) point dx'^k (dx^k, resp.) through the same number of lattice steps:

$$d\,(dy)^k \equiv -\overset{*}{\Gamma}{}^{k}_{mn}\, dx^m\, dy^n \equiv dx'^k - dx^k. \tag{3.14}$$

By definition, the lattice vectors in the intermediate state in this are of equal length and mutually parallel, which would correspond to an interpretation of A^k_κ as a lattice vector (cf., A.2). Correspondingly, one will get dx'^k from dx^k by displacing along dy^k by means of $\tilde{\Gamma}^k_{lm} = A^k_\kappa A^\kappa_{m,l}$, and as a result:

$$dx'^k - dx^k = -\,\tilde{\Gamma}^k_{lm}\, dx^m\, dy^n. \tag{3.15}$$

The affinity that is defined by the parallel displacement that was described above:

$$\overset{*}{\Gamma}{}^{k}_{mn} = \tilde{\Gamma}^k_{nm}, \tag{3.16}$$

is, in contrast to the state connection, neither torsion-free nor metric relative to g_{kl}, and as a result, that affinity cannot be identical to the state connection.

In particular, the RIEMANNian curvature tensor $\overset{*}{R}{}^{k}_{mnl}$ that is defined by means of $\overset{*}{\Gamma}{}^{k}_{mn}$ was introduced as a "density of holes tensor," so to speak, that described the density of point defects, which ZORAWSKI identified with the matter tensor. In analogy with the BURGERS circuit of a dislocation that is determined by the CARTANian torsion S_{mnk} (dislocation density), a point defect will be characterized by means of $\overset{*}{R}{}^{k}_{mnl}$ using a closed circuit of a vector B^m around a surface element dF^{mn} that yields a lack of closure:

$$dBk = \overset{*}{R}{}^{k}_{mnl}\, dF^{mn}\, B^l.$$

However, $\overset{*}{R}{}^{k}_{mnl}$ cannot be expressed in terms of only the quasi-plastic distortions in any state, and is therefore unsuitable for the description of point defects.

Appendix 1

Variation of the Lagrangian function

The variation of the kinetic energy yields the inertial term, as usual, and only the variation of the potential energy needs to be calculated. With:

$$\rho\,\delta U = \rho\frac{\partial U}{\partial \varepsilon_{\kappa\lambda}}\delta\varepsilon_{\kappa\lambda} = \sigma^{\kappa\lambda}A^{l}_{\lambda}\,\delta_{kl}\,\delta A^{k}_{\kappa} = -\,\sigma^{k}_{\kappa}\,\delta A^{\kappa}_{k}\,, \tag{1}$$

that will come down to varying the reciprocal lattice vectors A^{κ}_{k}. Due to (1.7), however, it is not δA^{κ}_{k}, but the variation of the density of point defects:

$$\delta A^{\kappa}_{K}\Big|_{X^{L}} = Q^{\kappa}_{K}\delta\hat{n}_{A} \tag{2}$$

that is the immediately relevant quantity ([1]). (The A^{κ}_{K} vary, despite the mass-point X^{L} being fixed, since the position of the point defect varies.)

A line of reasoning that is analogous to the derivation of the continuity equation will imply that:

$$\delta\hat{n}_{A} = -\,\partial_{L}\left(\hat{n}_{A}\,\delta X^{L}\Big|_{\hat{X}^{S}}\right) = -\,A^{l}_{L}\,\partial_{l}\left(\hat{n}_{A}\,\delta X^{L}\Big|_{\hat{X}^{S}}\right), \tag{3}$$

in which $\delta X^{L}\Big|_{\hat{X}^{S}}$ means the shift of the point defect "$\hat{X}^{S}$" along the segment δX^{L} in the system that moves with the matter. From now on, $\delta X^{L}\Big|_{\hat{X}^{S}}$ will be calculated from the variations δx^{l} ($\delta\hat{x}^{l}$, resp.) of the positions of the masses (point defect, resp.) in the rest system. That will make:

$$\delta X^{L}\Big|_{\hat{X}^{S}} = \delta X^{L}\Big|_{x^{l}} + \frac{\partial X^{L}}{\partial x^{l}}\delta x^{l}\Big|_{\hat{X}^{S}}\,. \tag{4}$$

The first term in this means the variation of the function $X^{L}(x^{l})$ for a fixed argument. The second term means the variation of the arguments. Analogously, for $X^{L}(x^{k}(X^{K}))$, $\delta X^{L}\Big|_{X^{K}} = 0$ will imply that:

$$\delta X^{L}\Big|_{x^{k}} = -\,\frac{\partial X^{L}}{\partial x^{l}}\delta x^{l}\Big|_{X^{K}}\,. \tag{5}$$

The quantities $\delta x^{l}\Big|_{X^{K}}$ ($\delta x^{l}\Big|_{\hat{X}^{K}}$, resp.) mean the shift in the mass-point (point defect, resp.) that is characterized by X^{L} ($\hat{X}^{L}$, resp.), which was denoted by δx^{l} ($\delta\hat{x}^{l}$, resp.) above. Therefore, one will get:

$$\delta A^{\kappa}_{K}\Big|_{X^{L}} = -A^{l}_{L}\,Q^{\kappa}_{K}\,\partial_{l}\left(\hat{n}_{A}\,A^{L}_{l}(\delta\hat{x}^{l} - \delta x^{l})\right) \tag{6}$$

for (2). $\delta A^{\kappa}_{K}\Big|_{X^{L}}$ can be converted into the desired $\delta A^{\kappa}_{K}\Big|_{x^{k}}$ with $A^{\kappa}_{K} = A^{\kappa}_{k}A^{k}_{K}$:

([1]) For the sake of clarity, the coordinates in the initial state will be denoted by upper-case letters in what follows.

$$\delta A_K^\kappa\Big|_{X^L} = A_K^k\,\delta A_k^\kappa\Big|_{X^L} + A_k^\kappa\,\delta A_K^k\Big|_{X^L}.$$

With

$$\delta A_K^k\Big|_{X^L} \equiv \delta\frac{\partial x^k}{\partial X^K}\Big|_{X^L} = \frac{\partial}{\partial X^K}\delta x^k\Big|_{X^L} = A_K^l\,\partial_l\,\delta x^k$$

and

$$\delta A_k^\kappa\Big|_{X^L} = \delta A_k^\kappa\Big|_{x^k} + \delta x^l\,\partial_l A_k^\kappa,$$

it will follow that:

$$\delta A_K^\kappa\Big|_{X^L} = A_K^k(\delta A_k^\kappa + \delta x^l\,\partial_l A_k^\kappa + A_l^\kappa\partial_k\delta x^l), \tag{7}$$

and with (1) and (6):

$$\delta U = -\sigma_m^k\,A_\kappa^m\,A_k^K\,A_L^l\,[\hat{n}_A\,Q_K^\kappa\,A_r^L(\delta\hat{x}^r - \delta x^r)]_{,l} + \delta x^l\sigma_\kappa^k\,\partial_l A_k^\kappa + \sigma_\kappa^k\,A_l^\kappa\partial_k\,\delta x^l. \tag{8}$$

After partial integration, when one considers the variation of ρ, in addition, one will get (up to outer surface terms that will vanish under volume integration):

$$\begin{aligned}\delta(\rho\,U) &= \delta x^r\,\partial_l\sigma_r^l + [(\sigma_\kappa^K\,A_L^l)_{,l}\,\hat{n}_A\,Q_K^\kappa A_r^L](\delta\hat{x}^r - \delta x^r)\\ &= \delta x^r\,\partial_l\sigma_r^l + [\sigma_{\kappa,r}^K\,\hat{n}_A\,Q_K^\kappa - \sigma_\kappa^K\,\hat{\Gamma}_{lr}^l\,\hat{n}_A\,Q_\kappa^K](\delta\hat{x}^r - \delta x^r),\end{aligned} \tag{9}$$

with $\hat{\Gamma}_{lr}^l$ as in (3.2).

The expression […] can be converted as follows: We regard Q_K^κ to be something that is assumed to be constant and the expression $Q_K^\kappa\sigma_\kappa^K$ to represent a scalar, so the partial derivative can be replaced with the covariant derivative that relates to the $\hat{\Gamma}$-affinity. Furthermore, we can get the density $\hat{n}$ of point defects in the final state from $\hat{n}_A$ upon multiplying by $\sqrt{\|\,b_{kl}\,\|}/\sqrt{\|\,a_{kl}\,\|}$, and finally, we define the transformation of the displacement dipole into the coordinate system of the final state by:

$$Q_m^n \equiv A_\kappa^n A_m^K\,\frac{\sqrt{\|\,a_{kl}\,\|}}{\sqrt{\|\,b_{kl}\,\|}}\,Q_K^\kappa. \tag{10}$$

$\|\,a_{kl}\,\|$ ($\|\,b_{kl}\,\|$, resp.) denotes the determinant of the covariant metric of the final (initial, resp.) state in that equation. If one considers (3.8) then one will get:

$$\hat{\nabla}_r\left(\sigma_n^m\,Q_m^n\,\frac{1}{\sqrt{\|\,a_{kl}\,\|}}\right)\hat{n}\sqrt{\|\,a_{kl}\,\|} - \sigma_n^m\,Q_m^n\,\hat{\Gamma}_{rl}^l\,\hat{n}$$

for the converted expression. When performing the covariant differentiation, it should be observed that the quantity to be differentiated represents a scalar density of weight – 1. It will ultimately yield the term that was given in (2.2).

Appendix 2

Distortions and lattice vectors

One can interpret the splitting of the distortions, when regarded as transformation matrices, as the image of the basis vectors in the original coordinate system; e.g., $\mathfrak{e}_\kappa$ will go to $A^k_\kappa \mathfrak{e}_k$, etc. For the sake of clarity and due to its physical meaning, it is preferable to connect the vector fields that are so defined with the lattice vectors [**8**].

Two possibilities must be distinguished in regard to the dependency of the lattice vectors on the character of the internal stress sources:

a) Plastic distortion:

The lattice vectors are all the same in the initial state, as well as the final one. One can then think of the intermediate state as something that arises by adding (removing, resp.) some elementary cells or by sliding without stress. That would correspond precisely to the insertion of dislocations.

b) Quasi-plastic distortion:

The lattice vectors are equal to each other only in the initial state. The intermediate state will then arise by distorting the building blocks of the lattice without stress; e.g., by inserting foreign atoms or extra matter, changes in temperature, or imposing electromagnetic fields.

Whereas the neighboring phenomena will be altered by plastic distortion (e.g., connecting vectors will not remain lattice vectors), the lattice vectors themselves will, however, remain invariant, and precisely the converse will first be true for quasi-plastic distortion.

If we now choose the system of lattice vectors to be the basis vectors in the initial and intermediate state, as the problem dictates, then as a result $A^k_\kappa \mathfrak{e}_k$ (for plastic distortion) or $A^k_K \mathfrak{e}_k$ (for quasi-plastic distortion), resp., will represent the lattice vectors in the final state. The parallel translation that is constructed from the field of lattice vectors will then be given by the lattice affinity:

$$\tilde{\Gamma}^k_{lm} = A^k_\kappa A^\kappa_{m,l} \qquad (\hat{\Gamma}^k_{lm} = A^k_K A^K_{m,l}, \text{ resp.}). \tag{A.2.1}$$

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