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Remark on our paper "On the kinematics and dynamics of the nonlinear continuum theory of dislocations"

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The argument that was presented in our paper [**1**] can be improved upon in two regards.

1. For the quantity of potential energy U, the significant variation is the one that the infinitesimal distance between two material points in a stressed state that differ by $dx^{(k)}$ will experience under elastic deformation:

$$ds^2 - d\sigma^2 = 2\, dx^{(k)}\, dx^{(k)}\, \varepsilon^{\text{el.}}_{(k)(e)}\,,$$

in which:

$$ds^2 = \delta_{ik}\, dx^i\, dx^k,$$
$$d\sigma^2 = \delta_{(i)(k)}\, dx^{(i)}\, dx^{(k)}.$$

Since $dx^{(k)}$ is fixed under that deformation, as opposed to dx^k or dx^K, the aforementioned physical state of affairs should be expressed by:

$$U = U\left(\varepsilon^{\text{el.}}_{(k)(l)}\right). \tag{1}$$

That corresponds to the process in the conventional theory by which the stressed reference state coincides with the so-called initial state K in the absence of dislocations. Due to (1), the material law is now written in the form:

$$\sigma^{(k)(l)} = \rho \frac{\partial U}{\partial \varepsilon^{\text{el.}}_{(k)(l)}}. \tag{2}$$

2. As was discussed before in **2.6**, the consideration of the kinematical auxiliary condition (1.3.4) under the variation of the vectors $A_k^{(k)}$ with the help of LAGRANGE multipliers raises certain doubts. We therefore propose to proceed along a path that is analogous to the way that the virtual displacements in point mechanics is restricted in the presence of scleronomic-anholonomic conditions. When we then regard the virtual

displacements δx^k and $\delta\tilde{x}^{(k)}$ of the particle and dislocation positions as possible displacement, we will place the same demands upon them as we would on the actual displacements $v^k\ \delta t$ ($\tilde{v}^k\,\delta t$, resp.). The arguments that led to the derivation of the fundamental kinematical equation can then be simply repeated and will yield:

$$\delta A_k^{(k)} + 2\,\delta\tilde{x}^p A_{[k,p]}^{(k)} + (A_p^{(k)}\delta x^p)_{,k} = 0$$

for the variation of the lattice vectors or also [when one considers that $\delta\left(A_k^{(k)}A_{(l)}^k\right) = 0$]:

$$\delta A^{(k)}{}_k + \delta x^n\,\partial_n A_{(k)}^k - A_{(k)}^k\,\partial_n \delta x^k + 2(\delta x^n - \delta\tilde{x}^n)A_{(k)}^e\,a_{ne}{}^k = 0. \qquad (3)$$

As was to be expected, that is the same expression for $\delta x^n = \delta\tilde{x}^n$ (viz., no displacement of the dislocations relative to the medium) that one finds for the variation of the deformation gradient in the conventional theory [cf., (2.3.5)], in which K can be formally replaced with (k).

With (1), (2), (3), the variation of the LAGRANGIAN function:

$$L = \tfrac{1}{2}\rho\, v_i\, v^i - \rho\, U\left(\varepsilon_{(k)(l)}^{\text{el.}}\right)$$

can now be performed more simply than in (2.5). The use of:

$$\varepsilon_{(k)(l)}^{\text{el.}} = \tfrac{1}{2}(A_{(k)}^k A_{(l)}^l \delta_{kl} - \delta_{(k)(l)})$$

will imply that:

$$\delta L = \delta x^k\left\{-\rho\frac{d}{dt}v_k + \sigma_{k,l}^{\,l} + 2\sigma_n^{\,i}\alpha_{ki}{}^n\right\} + \delta\tilde{x}^k\left\{-2\sigma_n^{\,i}\alpha_{ki}{}^n\right\}.$$

When only the position of the dislocation is varied $\delta\tilde{x}^k$, what will clearly arise here is the PEACH-KOEHLER force:

$$k_k = -\,2\sigma_n^{\,i}\alpha_{ki}{}^n\,,$$

as a nonlinear generalization of what happened in **2.6**. With that variational procedure, no additional term with the same type as the Lorentz force will appear along with the PEACH-KOEHLER force. As was to be expected, the arguments in the linear theory [**2**] can then be adapted to the nonlinear theory with no further discussion, in which the reversible motion of the dislocations is likewise determined by only the vanishing of the PEACH-KOEHLER force.

In conclusion, let it be pointed out that due to (1), for the sake of consistency, one should replace the PEACH-KOEHLER force (2.1.11) with (4) in the energy balance (2.1.10).

References

1. H.-A. BAHR and H. G. SCHÖPF, Ann. Phys. (Leipzig) **21** (1968), 57.
2. H.-A. BAHR, W. POMPE, and H. G. SCHÖPF, Phys. Stat. Sol. **28** (1968), K23.

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