"Berichtigung zu meiner Arbeit: 'Über die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie,'" Phys. Zeit. **22** (1921), 29-30.

Correction to my paper: "On the effect of rotating distant masses in Einstein's theory of gravitation." ([1])

By Hans Thirring

Translated by D. H. Delphenich

Von Laue and **W. Pauli** were kind enough to bring to my attention the following error in my aforementioned paper: For the approximate integration of **Einstein**'s field equations, the quantity dV in the formulas for the retarded potentials $\gamma_{\mu\nu}$ [eq. (3), *loc. cit.*] means simply the ordinary spatial volume element of the integration space (in polar coordinates: $r^2\,dr\,\sin\vartheta\,d\vartheta\,d\varphi$) and not, as I incorrectly asserted, the naturally-measured volume element:

$$i\,\frac{dx_4}{ds}\,r^2\,dr\,\sin\vartheta\,d\vartheta\,d\varphi\,.$$

One must therefore drop an overall factor of $i\,dx_4\,/\,ds$ in equations (10) and (12). Furthermore, in the transition from equation (12) to (13), it was incorrectly set: $\int\rho_0\,dV = M$. In place of that, the correct assignment would be:

$$\int\rho_0\,dV_0 = i\int\rho_0\,\frac{dx_4}{ds}\,dV = M,$$

or with consideration given to (11):

$$M = 4\pi\sigma a^2\left(1+\frac{\omega^2a^2}{3}\right).$$

Upon eliminating that mistake, one will get the following matrix [eq. (16)] for the coefficients $g_{\mu\nu}$ of the line element in place of (10).

$$g_{\mu\nu} = \left\{\begin{matrix} -1-\frac{2kM}{a}\left[1+\frac{a^2\omega^2}{3}-\frac{2\omega^2}{15}(z^2+x^2-2y^2)\right], & +\frac{2kM}{a}\frac{\omega^2}{5}xy, & 0, & +i\frac{4kM}{3a}\omega y \\ +\frac{2kM}{a}\frac{\omega^2}{5}xy, & -1-\frac{2kM}{a}\left[1+\frac{a^2\omega^2}{3}-\frac{2\omega^2}{15}(z^2+x^2-2y^2)\right], & 0, & -i\frac{4kM}{3a}\omega x \\ 0 & 0 & -1 & 0 \\ +i\frac{4kM}{3a}\omega y, & -i\frac{4kM}{3a}\omega x, & 0, & -1+\frac{2kM}{a}\left[1+a^2\omega^2-\frac{2\omega^2}{15}(2z^2-x^2-y^2)\right] \end{matrix}\right\}. \tag{16}$$

([1]) This Zeit. **19** (1918), pp. 33.

The equations of motion for the mass point will then read as follows:

$$\left.\begin{aligned}\ddot{x} &= -\frac{8kM}{3a}\omega\dot{y} + \frac{4kM}{15a}\omega^2 x,\\ \ddot{y} &= \frac{8kM}{3a}\omega\dot{x} + \frac{4kM}{15a}\omega^2 y,\\ \ddot{z} &= -\frac{8kM}{15a}\omega^2 z.\end{aligned}\right\} \tag{22}$$

The Coriolis force then remains unchanged from my original formulas (22), while the centrifugal force terms get multiplied by the factor 4/5.

Moreover, the equations of motion in the system of motion that rotates with angular velocity ω' [cf., eq. (25)] will now read:

$$\left.\begin{aligned}\ddot{x} &= 2\left[\omega'\left(1+\frac{2kM}{a}\right) - \omega\frac{4kM}{3a}\right]\dot{y} + \left\{\omega'^2\left(1+\frac{2kM}{a}\right) - \omega\omega'\frac{8kM}{3a} + \omega^2\frac{4kM}{15a}\right\}x,\\ \ddot{y} &= -2\left[\omega'\left(1+\frac{2kM}{a}\right) - \omega\frac{4kM}{3a}\right]\dot{x} + \left\{\omega'^2\left(1+\frac{2kM}{a}\right) - \omega\omega'\frac{8kM}{3a} + \omega^2\frac{4kM}{15a}\right\}y,\\ \ddot{z} &= -\frac{8kM}{3a}\omega^2 z.\end{aligned}\right\} \tag{25}$$

Here, as well, one corrects the error by simply multiplying the terms in ω^2 by the factor 4/5. As a result of a further error in calculation for the transformation from the rotating coordinate system, the factor 4/3 originally appeared in the term in $\omega\,\omega'$, instead of 8/3. Therefore, in the penultimate paragraph of my paper, two statements can be improved upon: "If one rotates the reference system in the same sense as the hollow sphere then the centrifugal force will initially decrease for small values of ω' and attain a minimum when ω' / ω is equal to the 'dragging coefficient.' From there on, it will increase again and attain the value that it originally had for $\omega' = 0$ as soon as ω' / ω is equal to twice the dragging coefficient."

The main result of my paper (viz., the appearance of centrifugal and Coriolis forces in the gravitational field of the distant rotating masses) remains completely unchanged.

Vienna, 15 October 1920.

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